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WEIGHT OPTIMUM ARCH STRUCTURES

by

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Thesis Advisor: David Salinas

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Weight Optimum Arch Structures

by

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ABSTRACT

This investigation is concerned with the optimization of arch structures. The DOT optimization code is used to minimize the volume of arch structures which are constrained by limits on stress, design geometry, and section dimensions. Modeling the arch structure by a series of bar-beam elements, the finite element method is used to compute element stresses. The DOT optimization code selects section dimensions to prevent failure due to element stresses exceeding the material yield stress. Specifically, through coordinate transformations between local element coordinates and global system coordinates the element stiffness matrices transform into the global stiffness matrix. The resulting system matrix equations are then solved for the system degrees of freedom, that is, displacements and slopes. The system degrees of freedom, in turn, are transformed back to the element level to compute the internal forces and moments and hence, the stresses. Results are presented for a number of cases with regard to optimization scheme and stress analysis.

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TABLE OF SYMBOLS

A	the cross-sectional area
b_i	width of the i^{th} element
c	the distance from the center line to the outmost fiber of the element; c=h/2
D	the Domain of the problem
DOT	*Design Optimization Tool* software from VMA Engineering
E	Young's Modulus of the arch material
$\underset{\sim}{f}^{i}$	the bar-beam force vector in the global coordinate system
$\underset{\sim}{f}^{i'}$	the bar-beam force vector in the elemental coordinate system
$\underset{\sim}{f}^{ai}$	the bar elemental force vector
$\underset{\sim}{f}^{bi}$	the beam elemental force vector
F	Concentrated axial force
$\underset{\sim}{F}^{a}$	the bar system force vector
$\underset{\sim}{F}^{b}$	the beam system force vector
$\underset{\sim}{F}^{A}$	the bar system force vector including the boundary term vector $\underset{\sim}{U}$
$\underset{\sim}{F}^{B}$	the beam system force vector including the boundary term vectors $\underset{\sim}{M}$ and $\underset{\sim}{V}$
FEM	Finite Element Method
$\underset{\sim}{G}$	the column vector of linear shape functions
h_i	height of the i^{th} element
I	cross-sectional moment of inertia
$\underset{\approx}{k}^{i}$	the bar-beam elemental stiffness matrix in x-y coordinates
$\underset{\approx}{k}^{i'}$	the bar-beam elemental stiffness matrix in local coordinates
$\underset{\approx}{k}^{ai}$	the bar elemental stiffness matrix in local coordinates
$\underset{\approx}{k}^{bi}$	the beam elemental stiffness matrix in local coordinates
$\underset{\approx}{K}$	the bar-beam system (global) stiffness matrix

$\underset{\approx}{K}^{A}$	the bar system (global) stiffness matrix
$\underset{\approx}{K}^{B}$	the beam system (global) stiffness matrix
l_i	length of the i^{th} element
L	the total length of the given structure
$\mathcal{L}$	the differential operator
M	Moment
M_{max}	Maximum Moment
M_o	Concentrated Moment
$\underset{\sim}{M}$	the moment boundary term vector
NEL	the total number of elements
$\underset{\sim}{P}$	the bar equation boundary term vector
p_x	axial loading
p_y	lateral loading
P	concentrated load
$\underset{\sim}{Q}$	the column vector of cubic shape functions
r	the ratio of the maximum shear stress to the normal stress due to bending; $r=\tau_{max}/\sigma_n$
R	the radius of the arch
R	the Residual function
s	the center-line coordinate of the arch
S_y	yield strength of the arch material
u	axial displacement
$\tilde{u}$	the approximate axial displacement
$\underset{\sim}{u}$	the vector of axial displacements
v	lateral "displacement"
$\tilde{v}$	the approximate lateral "displacement"
$\underset{\sim}{v}$	the vector of lateral displacements and slopes
V	the shear force
$\underset{\sim}{V}$	the shear force boundary term vector
x	the horizontal axis
y	the vertical axis
$\underset{\sim}{0}$	the zero vector
α_i	the angle the i^{th} element makes with the x-axis
β_i	the perpendicular compliment of α_i
$\underset{\sim}{\delta}^{i}$	the bar-beam displacement vector in the x-y coordinates
$\underset{\sim}{\delta}^{i'}$	the (6x1) bar-beam displacement vector assicated with $\underset{\approx}{k}^{i'}$

δ_{exact}	the exact analytical solution
Γ^i	the (6x6) local transformation matrix
$\tilde{\theta}$	the subtended arc of the arch
σ_a	the normal stress due to bar (axial) behavior
σ_b	the normal stress due to beam (bending) stress
σ_i	the maximum stress developed in the i^{th} element
σ_n	the total normal terss
τ_{max}	the maximum shear stress

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I. INTRODUCTION

A. BACKGROUND

Over 5,000 years ago, evolution of the post and lintel structures of the stone age gave rise to the arch. Highly regarded for its graceful shape and design suitability, the simple arch structure has been applied to engineering and architectural designs ever since. The ancient Roman Coliseum and aqueducts, great cathedrals of the Middle Ages, and railway bridges of modern history are just a few of the many examples of structures comprised of arches standing today (Figure 1.1). Throughout its history, engineers and architects have labored to improve the design of the arch in order to enhance the overall design structure. This desire for perfection has led engineers to devise a rational, directed design procedure and hence, the concept of optimization was created.

The advent of the computer era has lead to 20 years of extensive development in the use of numerical optimization techniques. These techniques offer a logical approach to design decisions where intuition and experience previously prevailed. Coupled with trends toward material and cost efficiency, numerical optimization has prompted considerable research in the field of automated design [Ref. 1]. As

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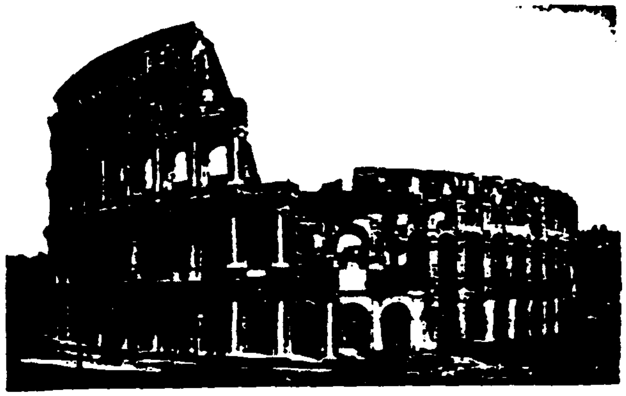

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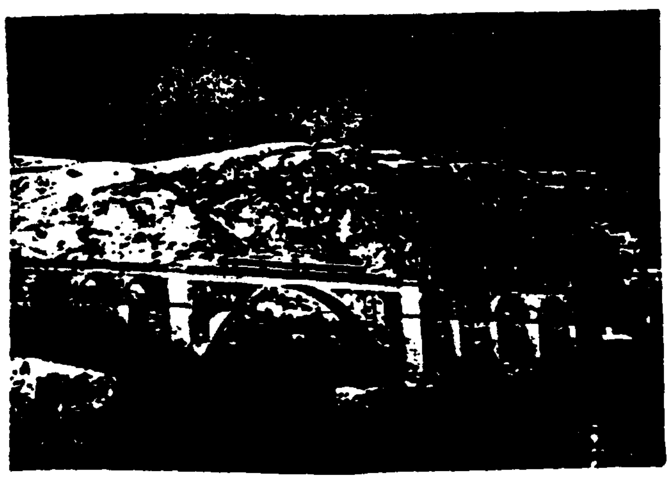


Figure 1.1 (a) The Coliseum
(b) Firth of Forth Railway Bridge
(c) Isernia, Italy, railway bridge

Photos by Mrs. P. Menzies and CDR D.C. Warner

a step in design optimization of structures, the arch has been the subject of numerous optimality studies to enhance applicability in engineering and architectural designs.

One such study was performed by Farshad in 1976 [Ref. 2]. Using calculus of variations, he derived optimality conditions for nonlinear partial differential equations for hinged-hinged arches. The total potential energy of the system, augmented with several objective functions via Lagrange multipliers, was minimized with respect to design and state variables to achieve equilibrium and optimality. The nonlinear systems of equations for optimal thrust, minimum length of the arch, and minimum volume were presented but not solved.

In 1980, Rozvany *et al.* [Ref. 3] used the Prager-Shield criteria to optimize statically determinate arches. His 'arch' consisted of two inclined funicular frame beams ridgedly interconnected with a concentrated load applied at the joints. In the optimal 'arch' only bending or axial forces develop depending on the ratio of 4L/D, where L is the span of the structure and D is the depth of the cross section. Ratios greater than eight to one produced axial forces only and the optimal shape has a height of half the span. Ratios smaller than eight to one develop only bending and the optimum structure is a straight beam. In each case, the width of the beam segments for the optimal 'arch' varied linearly from the hinged support to the axis of symmetry.

That same year, Lipson *et al.* [Ref. 4] used the 'complex' method to optimize parabolic arches subject to uniform loading. His 'arch' was comprised of equal length straight beam sections of thin walled rectangular tubes. Maintaining constant depth and width for each segment, the vertical and horizontal wall thicknesses determined the arch shape which was optimized for minimum total weight. An arch with a rise of 0.342 times the span length proved to be the optimum.

In 1988, Ang *et al.* [Ref. 5] solved the arch optimization problem by parametricing the unspecified arch axis using spline functions and employing a smoothing function to approximate the non-smooth objective function. The 'arch' was considered to be a 'plastic' design of rectangular cross section subject to bending and axial compression. Three types of boundary conditions were imposed, simply supported-simply supported, clamped-clamped, and simply supported-clamped. The optimum shape of the arch is claimed to be a parabola with a rise of 0.433 times the span length. Apparently, there is some disagreement between these results and those previously noted.

In addition to arch optimization studies, Ding and Esping [Ref. 6] solved the minimum weight design problem for frame structures when stress and displacement constraints are considered. Using dual numerical methods, seven cross-sectional shapes were treated by approximating the stresses with pseudo and virtual load techniques. Results were

presented for a beam clamped at both ends, a portal frame, a 2 X 5 grillage, and a helicopter tail boom structure. Although Ding and Esping's investigation does not specifically solve for arch structures, the approximations used are completely detailed with convincing results.

In December of 1990, Charles Scott McDavid of the Naval Postgraduate School presented his thesis, "Weight Optimum Arch Structures," which optimized circular arches subject to various loadings and end conditions. Specifically, he optimized arches segmented into rectangular boxes that varied in width only. Through his research he concluded that a bar/beam element model is a viable technique for the approximation of arch structures, and that an arch structure that is more statically indeterminate is more efficient under identical loading. Additionally, he proposed possibilities for future researcn which includes varying both the height and width dimension, the major thrust of this investigation.

B. PROBLEM DEFINITION

In order to provide an in depth study, each of the cited investigations began with a problem definition and specific assumptions about the type of arch to be considered. For this investigation, the arch is defined as a structure of constant curvature (i.e., circular arches) which when supported at both ends and loaded laterally develops perpendicular reactions. This is intended to eliminate thick walled curved beams and

straight beams which develop virtually no perpendicular reactions when loaded laterally. Additionally, the cross-section dimensions are small relative to the radius of curvature and therefore the centroidal and neutral axes are assumed to coincide. Without the thin depth assumption, complications arise in the calculations of the displacements and the slopes because the arch no longer behaves as predicted by the beam equilibrium equation:

$$(EIv'')''=P_y(s) \tag{1.1}$$

and the bar equilibrium equation:

$$(AEu')'=-P_x(s) \tag{1.2}$$

where the prime superscript notation denotes differentiation with respect to the independent variable, s, and

E	=	Young's Modulus
I	=	Cross-sectional Moment of Inertia
v	=	Lateral Displacement
P_y	=	Lateral Loading
A	=	Cross-sectional Area
u	=	Axial Displacement
P_x	=	Axial Loading
s	=	the Independent Variables

In order to facilitate the development of a finite element code to approximate the local displacements, the arch is approximated by a series of straight segments. From the local displacements, the virtual load techniques, as described in the Ding and Esping paper, are applied to determine the internal psuedostresses. Once the stress distribution is determined, the arch volume is minimized to a structure that

maintains the developed stresses below the predefined maximum allowable stress.

The thrust of this investigation is to minimize the total weight of a linearly elastic, isotropic, and homogeneous arch under a variety of loadings and end conditions. Optimization in this investigation refers to the variance of the cross-sectional dimensions (that is, the design variables) to obtain optimum least weight structures. Design Optimization Tool (DOT) software [Ref. 7] is used to perform the optimization subject to prescribed constraints on the design variables as well as on the stress limitations. The objective is to minimize the total volume of the arch while maintaining stresses below the yield strength of the arch material. The intent of this study is to provide direction and guidance on which further research for weight optimization may be developed.

II. PROBLEM FORMULATION

A. PROBLEM STATEMENT AND ASSUMPTIONS

As noted in the introduction, the purpose of this investigation is to optimize arch structures to form a foundation upon which further research can be based. These arch structures, subject to specified loadings and end conditions, vary in cross sectional geometry to minimize the weight. In order to limit the scope of this study, approximations and specific assumptions are made as follows:

- The arch maintains a constant radius of curvature.
- The arch is approximated by a series of straight segments of a solid rectangular cross sectional geometry.
- Cross section design is restricted to ensure the applicability of beam and bar equilibrium equations (1.1 and 1.2).
- To prevent failure the internal stresses developed due to the loading must not exceed the yield strength of the material.
- The arch structure is composed of a linearly elastic, isotropic, homogeneous material.

To begin the design optimization process, the arch structure is approximated by contiguous straight line segments. Each segment is modeled by a bar-beam structure connects to the adjacent segment at a point defined as the nodal point. At each nodal point, the cross section base and

height dimensions are selected as the design variables. From this model, the optimization problem can be formulated into objective and constraint functions which are functions of these design variables.

B. MATHEMATICAL MODEL

Due to the complex nature of this problem, the constant radius arch structure is modeled by a series of straight contiguous elements where the arch radius of curvature, R, and the number of elements used to approximate the arch, NEL, is specified. (Figure 2.1) For simplicity, the length of each element is constant such that:

$$L = \theta R / NEL$$

where θ represents the subtended arc of the arch.

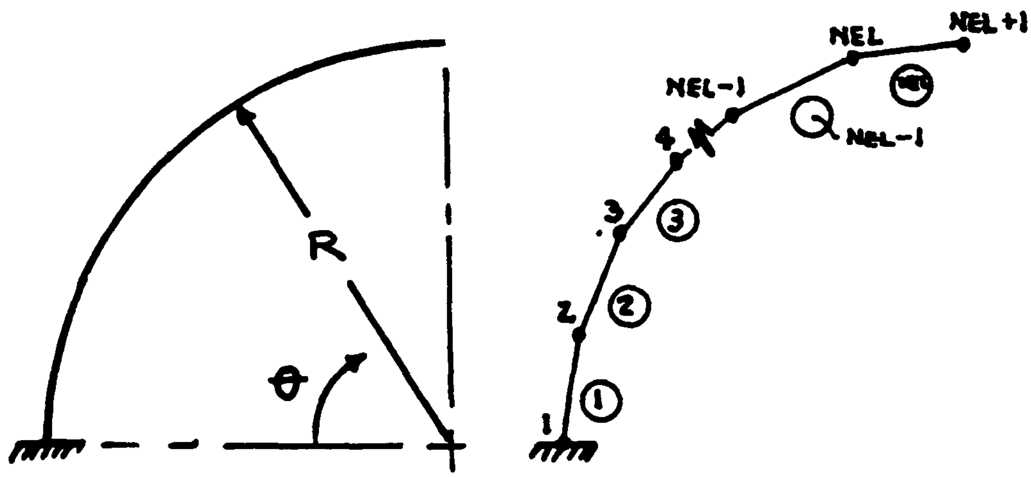


Figure 2.1 Arch Structural Model

At each nodal point, there exists a base and height dimension such that the cross sectional dimensions from one element to the adjacent element maintains smooth piecewise continuity. (Figure 2.2) The resultant element shape is that of a three dimensional trapezoid whereby the volume is calculated by multiplying the average base and height with the length of the element. In mathematical terms, the volume of the ith element is calculated as follows:

$$Volume(i) = B_{ave}(i) * H_{ave}(i) * L \tag{2.1}$$

where

$$B_{ave} = (B\ (i) + B\ (i+1))/2 \tag{2.2}$$

$$H_{ave} = (H\ (i) + H\ (i+1))/2 \tag{2.3}$$

B = the Nodal Base Dimension
H = the Nodal Height Dimension
L = the Element Length
i = the ith Element

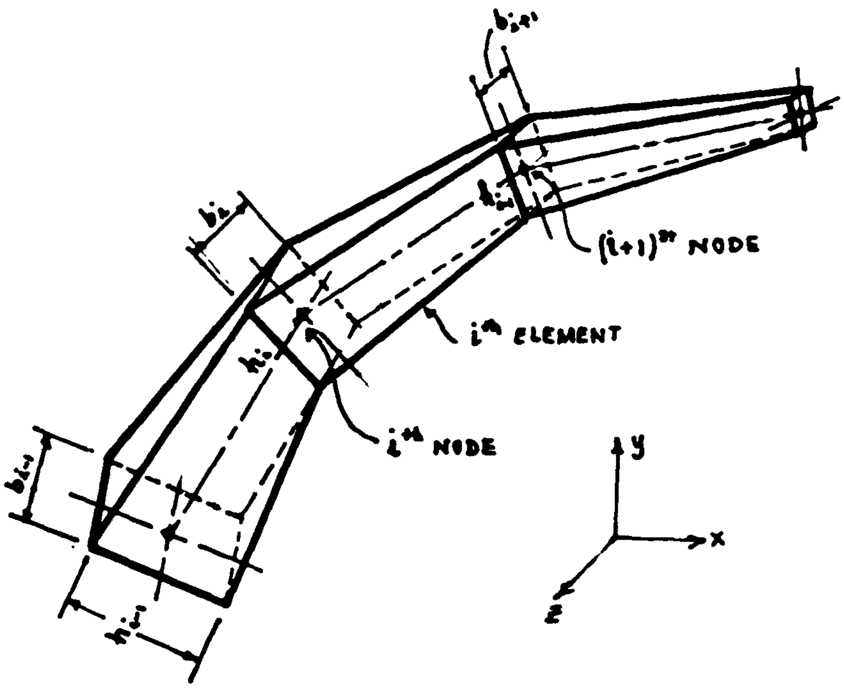


Figure 2.2 Arch Elements

Defined in the problem statement, the optimal arch is achieved by varying the cross sectional dimensions, the base and height, in order to minimize the weight. Thus the nodal base and height dimensions are the design variables for which the objective function is defined.

C. OPTIMIZATION PROBLEM

The objective of this study is to minimize the weight of an arch structure while maintaining a stress distribution which does not exceed the yield strength of the material. Additionally, other constraints on the design variables are imposed. Since the arch is composed of a homogenous material, the weight of the arch is directly proportional to the volume of the arch. Thus, the objective of this investigation is satisfied by minimizing the total arch volume. The total arch volume, Vtot, is the sum of the elemental volume, v(i). Thus in mathematical form, the objective function is as follows:

$$Objective = MIN\,(V_{tot}) = \left\{ MIN \sum_{i=1}^{NEL} v\,(i) \right\} \qquad \textbf{(2.4)}$$

where the elemental volumes, v(i), calculated by Equation (2.1), is summed for all elements to compute the total arch volume.

In keeping with the assumptions made in the problem statement, the objective function is constrained in order to impose practical and important physical restriction on the

problem. Properly defined, the constraints are used to avoid undesirable behavior such as yielding, to ensure validity of the governing equilibrium equations, and to provide a realistic design. For this study, the constraints fall into three categories, strength criteria, geometric limitations, and side constraints.

First, for specified loadings and end conditions, the optimized arch must not 'fail by yielding.' Assuming the arch material to be linearly elastic, the applied loading must not cause the structure to exceed the elastic limit of the selected material. Therefore, the internal stresses developed must remain below the yield strength of the material. Mathematically, the strength criteria is as follows:

$$\sigma(i) \leq Sy$$

or in normalized form:

$$(\sigma(i)/Sy) - 1.0 \leq 0.0 \qquad \textbf{(2.5)}$$

where $\sigma(i)$ is the maximum stress developed at the ith nodal point of the arch and Sy is the yield strength of the arch material selected by the designer. Unfortunately, the stress distribution, in terms of the design variables is not readily available. However, using the beam and bar equilibrium equations (1.1 and 1.2), a finite element scheme based on the model can be developed to determine the arch's displacements and slopes due to a given loading. Knowing how the

displacements and slopes change throughout the arch, the stresses at the nodal points can be calculated.

Secondly, limits must be imposed on the cross sectional geometry in order to ensure applicability of the bar and beam equilibrium equations. Limiting the cross section base and height dimensions relative to one another prevents the structure from becoming either a shell-like or deep curved beam structure. To maintain the geometry of the arch, the following conditions are imposed:

$$B(i) - 3.0 * H(i) \leq 0.0 \tag{2.6}$$

and

$$H(i) - 10.0 * B(i) \leq 0.0 \tag{2.7}$$

Finally, the side constraints are imposed to ensure a realistic solution. The arch is a physical object that must have a realistic finite cross sectional area; however, these section dimensions must also remain small relative to the radius of curvature by definition of the arch. Thus, the side constraints for the base and height dimensions are as follows:

$$0.03\,in. \leq B(i) \leq 6.0\,in. \tag{2.8}$$

$$0.03\,in. \leq H(i) \leq 6.0\,in. \tag{2.9}$$

In the future, additional constraints should be considered such as global buckling and local crippling.

III. OPTIMIZATION ANALYSIS

To perform the computer optimization, the *Design Optimization Tools* (DOT) software package is used due to its availability, user friendliness, and reputation. DOT, a FORTRAN 77 optimization software package available from VMA Engineering, uses numerical search methods to seek a minimum value of one function, the objective, subject to the limits of others, the constraints [Ref. 7]. DOT has two methods for iteratively solving constrained optimization problems, the Modified Method of Feasible Directions and the Sequential Linear Programming Method.

A. MODIFIED METHOD OF FEASIBLE DIRECTIONS

Modified Method of Feasible Directions is a numerical method that deals directly with nonlinear problems. For this method, a search direction vector, $\underline{S}$, is first found. The design point is then moved in this direction to update the design variable vector, $\underline{X}$, according to the equation:

$$\underline{X}_q = \underline{X}_{q-1} + \alpha^* \underline{S}_q \tag{3.1}$$

where the scaler quantity α^* defines the distance moved in the $\underline{S}$ direction, and q represents the iteration number.

For an initial design, say $\underline{X}_o$, the design is moved in the direction of the steepest descent until a constraint is encountered.

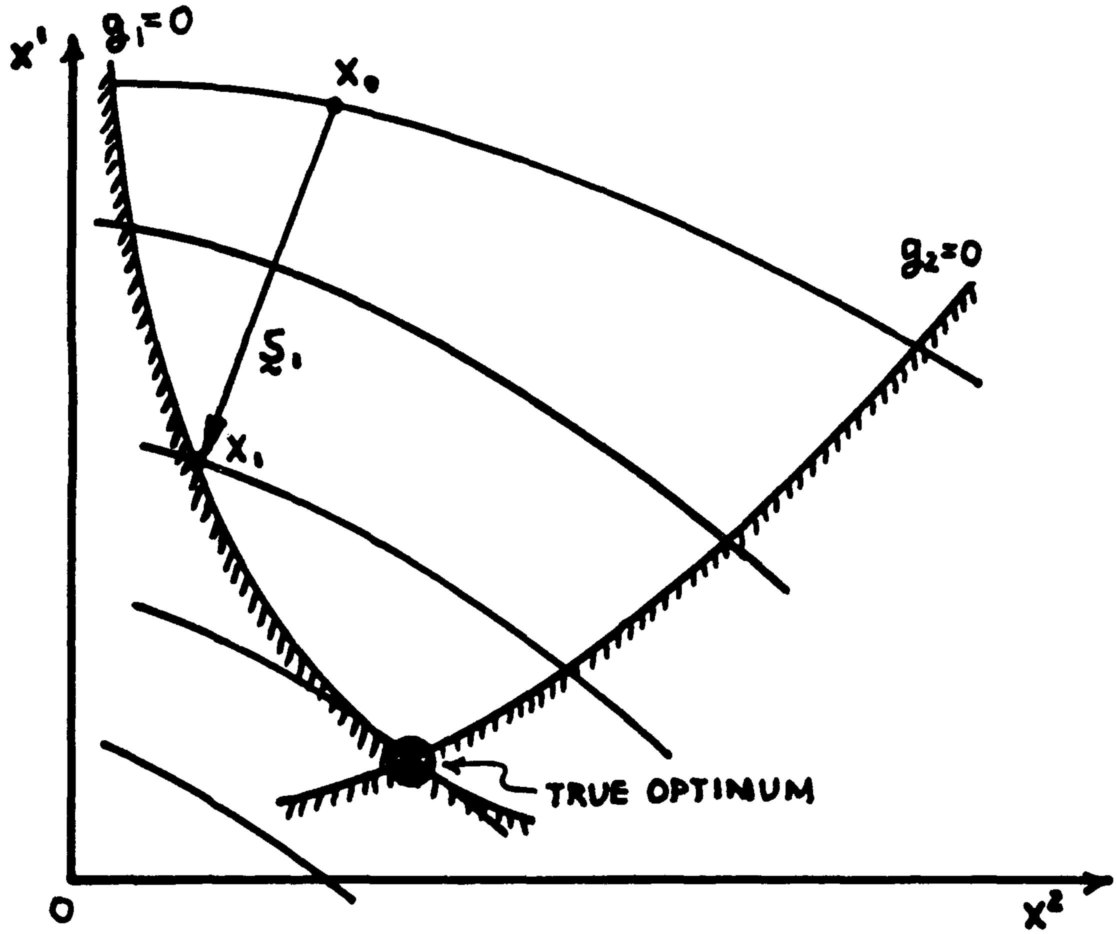


Figure 3.1 Modified Method of Feasible Directions: The Search Direction

Having encountered the constraint boundary, a new search direction is found by solving the subproblem:

Maximize:

$$MAXIMIZE: \quad \underline{p} \cdot \underline{Y} \qquad (3.2)$$

Subject to:

$$\underline{\underline{A}}\,\underline{y} \leq 0 \tag{3.3}$$

$$y \cdot \underline{y} \leq 1 \tag{3.4}$$

where

$$\underline{y} = \begin{Bmatrix} S_1 \\ S_2 \\ \vdots \\ S_n \\ \beta \end{Bmatrix} \tag{3.5}$$

$$\underline{p} = \begin{Bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{Bmatrix} \tag{3.6}$$

$$\underline{\underline{A}} = \begin{bmatrix} \nabla^T g_1(\underline{x}) \\ \nabla^T g_2(\underline{x}) \\ \vdots \\ \nabla^T g_j(\underline{x}) \\ \nabla^T F(\underline{x}) \end{bmatrix} \tag{3.7}$$

The search direction, $\underline{S}$, will follow the constraint yet allow the design to leave a constraint boundary if the objective will reduce farther. In general, the form for inequality constraint problems is:

Maximize:

$$-\nabla F(\underline{x}) \cdot \underline{S} \tag{3.8}$$

Subject to:

$$\nabla g_j(\underline{x}) \cdot s \leq 1 \qquad j \in J \tag{3.9}$$

$$\underline{S} \cdot \underline{S} \leq 1 \tag{3.10}$$

When the search direction is away from a currently active constraint and the scaler product of the gradient of each critical constraint with the $\underline{S}$ vector is less than zero, the constraint is omitted from the set of active constraints. If $\underline{S}$ is the null vector or numerically small, the optimization process is terminated because the Kuhn-Tucker conditions for optimality have been met.

B. SEQUENTIAL LINEAR PROGRAMMING

The second numerical method, Sequential Linear Programming (SLP), linearizes nonlinear objective and constraint functions and then obtains a solution using linear programming methods. Once the approximate solution is found, the functions are linearized about the new design point and the a linear programming problem approximated and solved. By repeatedly linearizing and solving the resulting problem, a precise solution is achieved.

In general format, the nonlinear functions are linearized via a first-order Taylor series expansion as follows:

Minimize:

$$F(\underline{x}) = F(\underline{X}_o) + \nabla F(\underline{X}_o) \cdot \delta \underline{X} \tag{3.11}$$

Subject to:

$$g_j(\underline{x}) = g_j(\underline{X}_o) + \nabla g_j(\underline{X}_o) \cdot \delta \underline{X} \leq 0 \qquad j=1,m \tag{3.12}$$

where

$$\delta \underline{X} = \underline{X} - \underline{X}_o \tag{3.13}$$

and the zero subscript identifies the point about which this Taylor series expansion is performed. At the initial design, $\underline{X}_o$, the objective and constraints are linearized to give straight line representations of the functions.

Typically, this method converges to the optimum solution with fewer iterations than the previous method mentioned. However, as seen in Figure 3.2, the optimum of the approximated linear problem is infeasible (i.e., a design that violates some or all of the constraints). Additionally, certain linearizations produce unbounded linear problems. However, imposing move limits on the linear approximation helps ensure that the optimum will eventually be reached.

C. DOT PROGRAM PARAMETERS

For both numerical methods, there are several parameters that can be adjusted within DOT in order to 'fine tune' the program for a specific problem. Fine tuning is a process in which the program parameters are internally adjusted to

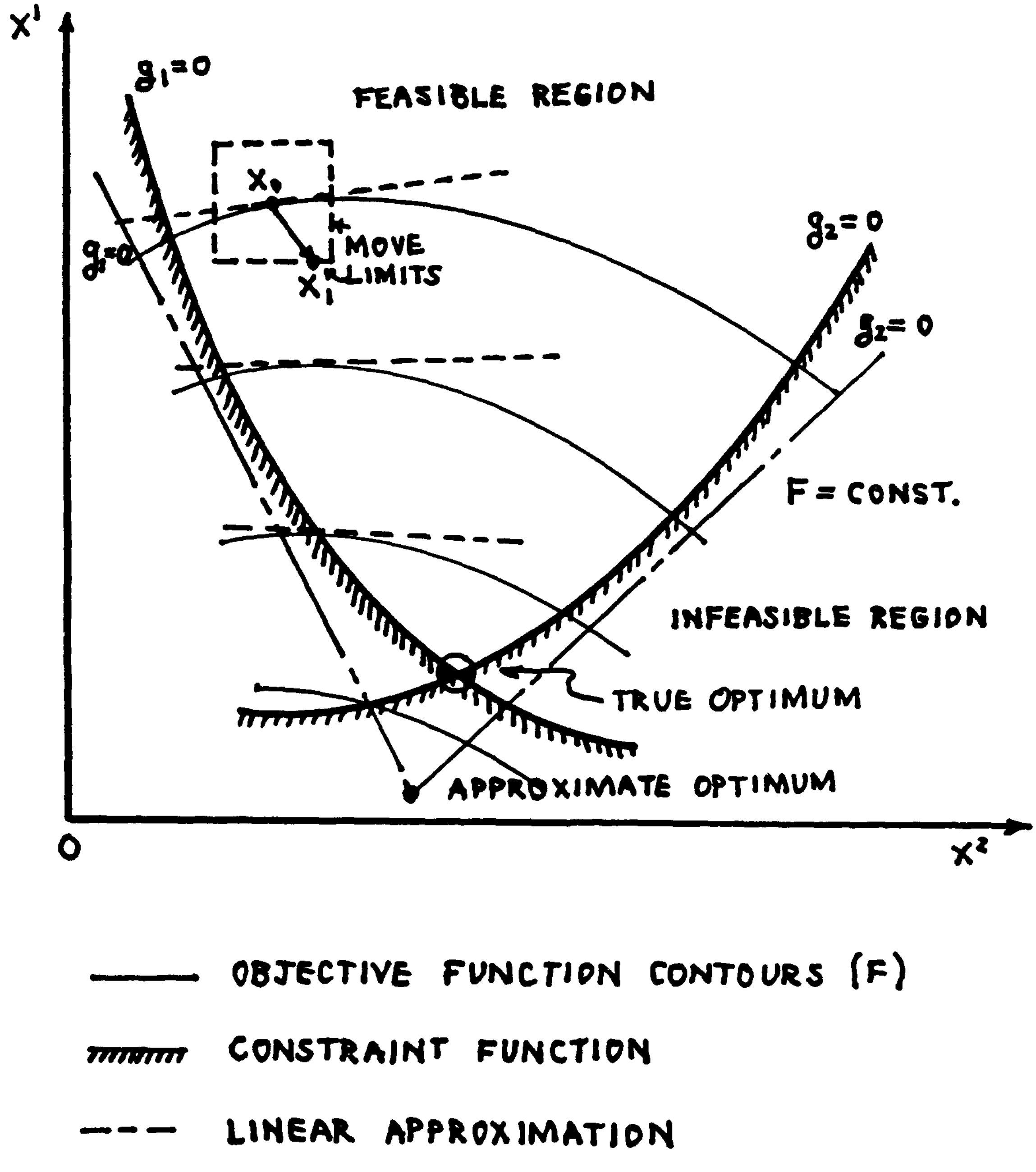


Figure 3.2 Sequential Linear Programming: The Linearized Problem

optimize the optimizer performance. With proper tuning, the optimization process can be designed to remain within specified tolerances and operate more efficiently. A complete listing of all the DOT parameters is contained in Appendix A. However, for the purpose of this investigation, only the constraint boundaries, auto scaling, and termination tolerance parameters were tuned to enhance optimization performance. [Ref. 7]

First, for constrained optimization, the constraint boundary must be established. Mathematically defined, the constraint is considered active if its numerical value is between the value of CT and CTMIN, and violated if its numerical value is greater than CTMIN. By using a narrow band to approximate the constraint function, the optimizer is less likely to exceed convergence criteria without achieving an optimal design. In the realm of design, CTMIN is of particular concern. Principally, it is a small positive number that controls how far the design can deviate from the constraint boundaries and still be considered a feasible design. In theory, CTMIN can be reduce to zero to avoid any constraint violations, however, it is not practical due to the large number of iterations and computer expense required.

In addition, it is normally considered good engineering practice to normalize design variables and nondimensionalize basic parameters [Ref. 1]. For optimization, variables are scaled to affect normalizing by evaluating the diagonals of

the Hessian matrix of the objective and constraint functions. As the optimization proceeds, reevaluation is sometimes necessary to rescale the variables. The DOT parameter ISCAL may be selected to rescale the design variables over an interval or eliminate the scaling function all together. Unfortunately, the DOT manual indicates that there is no established theory for scaling. Scaling is therefore a function of trial and error.

Last, the termination criteria also has a major effect on the efficiency and reliability of the optimization process. Termination criteria is established so that the design process is stopped when the number of iterations exceeds a specified limit. DOT parameters ITMAX and JTMAX specify the maximum number of iterations allowed for the Modified Method of Feasible Directions and the SLP method respectively. This ensures that the program will not iterate indefinitely. Furthermore, the progress of the optimization is checked for convergence. Design convergence is achieved when the change in the value of the objective function from one iteration to the next approaches zero. The DABOBJ parameter is a specified tolerance for which the maximum absolute change in the objective function between iteration is numerically small. Additionally, ITRMOR and ITRMST are parameters which specify the number of consecutive iterations for which the design change is less than DABOBJ for Modified Method of Feasible Directions and the SLP method respectively.

IV. STRESS ANALYSIS

The objective of this investigation is to minimize the total weight (volume) of a load bearing arch subject to specified constants. To obtain an optimal structure, DOT is interfaced with an analysis program which computes the values of the objective and constraint functions in terms of the design variables, specifically the cross sectional dimensions. Since the strength constraint requires that the stresses at any point do not exceed the yield strength of the arch material, the stress distribution over the domain of the arch must be known. However, as indicated in Chapter II, the stress distribution is not readily available in terms of the cross sectional dimensions. Therefore the following stress development is pursued for optimization.

A. STRESS DEVELOPMENT

For this study, the strength constraint requires that the applied load will not cause the arch to fail by yielding. Therefore, the internal stresses developed must remain below the yield strength of the material. For this study, the stresses considered are composed of normal stresses due to bending moments and axial forces where the total normal stress is the algebraic sum of these components expressed as follows:

$$\sigma_n = \sigma_b + \sigma_a \quad \textbf{(4.1)}$$

where σ_n = total normal stress
σ_b = the normal stress due to bending
σ_a = the normal stress due to axial force

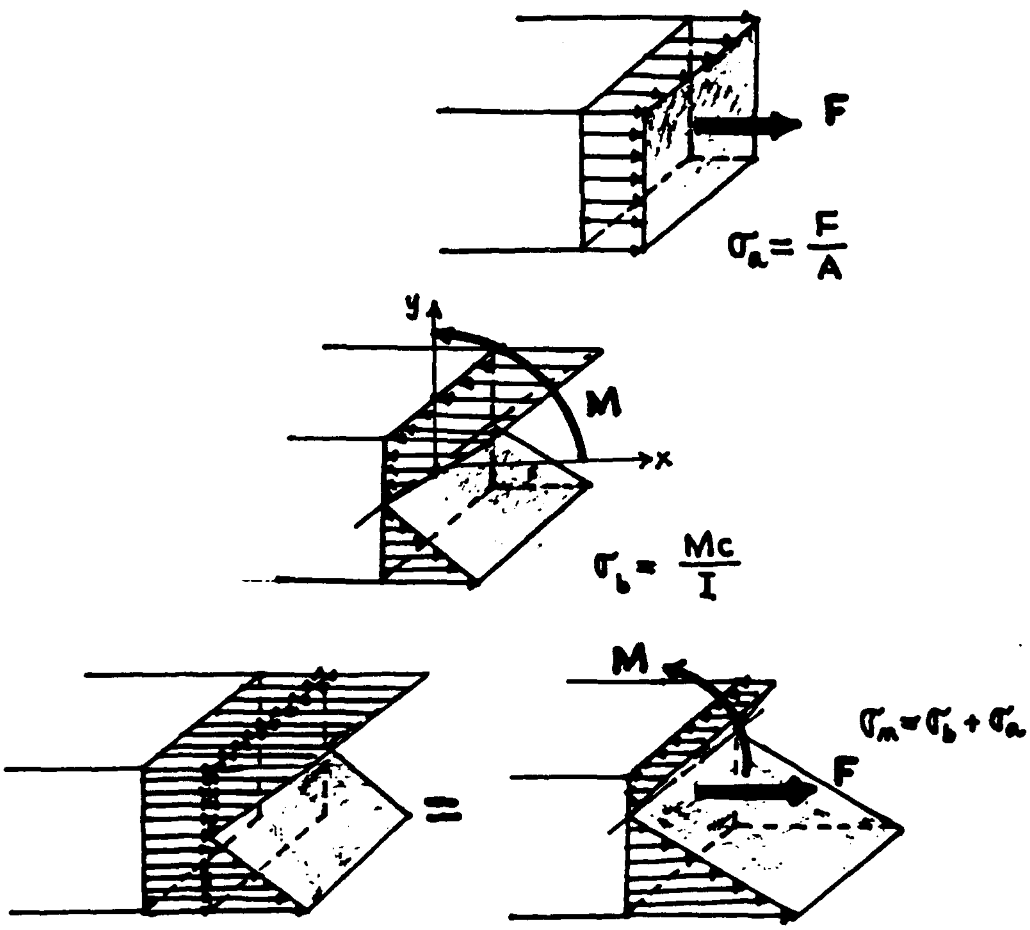


Figure 4.1 Normal Stresses Due to Bending Moments and Axial Forces

Shear stresses may also develop within the arch from shearing forces; however, the side constraints limit the geometry such

that these stresses are negligible. (See Appendix B for the complete justification for Shear stress omission.)

To compute the two normal stress components, the arch is sectioned and approximated by straight frame elements. Thus, the stresses can be determined for each element endpoint (or nodal point) in order to establish the stress distribution. Each element is considered to behave as both a tapered beam, to calculate the stresses due to bending, and a tapered bar, to calculate stresses due to axial forces.

First, for a straight beam segment, the maximum normal stress due to bending, hereafter referred to as bending stresses, is defined by the following equation:

$$\sigma_b = \frac{Mc}{I} \tag{4.2}$$

where c is the distance from the neutral axis to the point furthest from the neutral axis. The moment, M, at a section is calculated by:

$$M = EIv'' \tag{4.3}$$

resulting from the beam equilibrium equation (1.1). With substitution and simplification, Equation (4.2) becomes:

$$\sigma_b = ECv'' \tag{4.4}$$

In the same manner, the normal stress due to axial behavior is determined. For a bar element, the normal stress due to axial forces, hereafter referred to as axial stresses, is defined by the equation:

$$\sigma_a = \frac{F}{A} \tag{4.5}$$

where A is the cross section area and the axial force, F, is calculated by:

$$F = AEu' \tag{4.6}$$

resulting from the bar equilibrium equation (1.2). Again, substituting and simplifying, Equation (4.5) becomes:

$$\sigma_a = Eu' \tag{4.7}$$

Final substitution into Equation (4.1) results in an equation for total normal stress as follows:

$$\sigma_n = E(cv'' + u') \tag{4.8}$$

where Young's Modulus of elasticity, E, is a function of material selection, the distance from the neutral axis to extreme fiber, c, is a function of cross section height, and u' and v'' are the first and second derivatives of axial and lateral displacements respectively. Using the Galerkin Finite Element Method, approximate values for the axial and lateral displacements can be determined at element endpoints. From these values, the stress distribution is computed and the optimization process can proceed.

B. THE FINITE ELEMENT BEAM EQUATION DEVELOPMENT

The Galerkin Finite Element Method (FEM) is an approximation method which transforms a linear differential

equation into a system of linear algebraic equations. Using the beam equilibrium equation (1.1), approximate lateral displacements for the arch can be determined at the system nodal points. For this method, a family of hermite cubic shape function which possess the Kronecker Delta property, are introduced in order to maintain the necessary function and slope continuity for the fourth order beam equation. An approximate solution, $\tilde{v}$, for displacement, V, is formed as follows:

$$v \approx \tilde{v} = \underline{Q}^T \underline{V} \tag{4.9}$$

where v is the exact solution of the beam equation in continuous space, $\tilde{v}$ is the approximate solution in discrete space, $\underline{Q}^T$ is the transpose of a column vector of the cubic shape functions, and $\underline{V}$ is the vector of lateral displacements and slopes.

After the approximation is fcrmulated, the next step in the Galerkin method is to form the residual, R, in the following format:

$$R = \mathfrak{L}(\tilde{v}) - p_y(s) \tag{4.10}$$

where p_y is the lateral excitation force and $\mathfrak{L}$ denotes the differential operator which in the case of the beam equilibrium equation is defined by:

$$\mathfrak{L}(v) = [EI(v'')]'' \tag{4.11}$$

With substitution, the residual becomes:

$$R = [EI(\underline{Q}^T\underline{v})'']'' - p_y(s)$$

From the residual, the Galerkin Equations are formed:

$$\int_D \underline{Q}(R)\, ds = \underline{0} \tag{4.13}$$

where 0 is the null vector. Further substitution for R into the Galerkin vector equation results in :

$$\int_D \underline{Q}\,[EI(\underline{Q}^T\underline{v})'']''ds - \int_D \underline{Q}p_y(s)\,ds = \underline{0} \tag{4.14}$$

To solve the Galerkin Equation, integration by parts is performed twice which yields:

$$\begin{gathered} \underline{Q}[EI(\underline{Q}^T\underline{v})'']'|_B - \underline{Q}'EI(\underline{Q}^T\underline{v})''|_B \\ + \int_D \underline{Q}''EI(\underline{Q}^T\underline{v})''ds - \int_D \underline{Q}p_y(s)\,ds = \underline{0} \end{gathered} \tag{4.15}$$

where $|_B$ denotes evaluation of these vectors at the boundary points of the structure. Recognizing that the lateral displacement and slope vector is constant, Equation 4.15 is rewritten as:

$$\begin{gathered} \underline{Q}[EI(\underline{Q}^T)'']'\underline{v}|_B - \underline{Q}'EI(\underline{Q}^T)''\underline{v}|_B \\ + \int_D \underline{Q}''EI(\underline{Q}^T)''ds\underline{v} - \int_D \underline{Q}p_y(s)\,ds = \underline{0} \end{gathered} \tag{4.16}$$

From the beam equilibrium Equation (1.1), the shear, V, is defined by:

$$V = EIv''' \tag{4.17}$$

and Moment, M, by:

$$M = EIv'' \tag{4.18}$$

Thus, the boundary term load vectors are defined by:

$$\underline{V} = \underline{Q}[EI(\underline{Q}^T)'']'\underline{v}|_B \tag{4.19a}$$

and

$$\underline{M} = \underline{Q}'EI(\underline{Q}^T)''\underline{v}|_B \tag{4.19b}$$

Additionally, for convenience a system stiffness Matrix, $\underline{\underline{K}}^B$, is defined by:

$$\underline{\underline{K}}^B = \int_D \underline{Q}''EI(\underline{Q}^T)''ds \tag{4.19c}$$

and a system Force vector, $\underline{F}^b$, by:

$$\underline{F}^b = \int_D \underline{Q}p_y(s)\,ds \tag{4.19d}$$

Substitution of Equations (4.19 a through d) into Equation (4.16) results in the following system of linear algebraic equations:

$$\underline{V}|_B - \underline{M}|_B + \underline{\underline{K}}^B\underline{v} - \underline{F}^b = \underline{0} \tag{4.20}$$

Further simplification is possible by defining F^b as the load vector of internal and external applied lateral loads by:

$$\underline{F}^B = \underline{F}^b + \underline{M}|_B - \underline{V}|_B \tag{4.21}$$

Thus, Equation (4.20) reduces to:

$$\underline{\underline{K}}^{B}\underline{v}=\underline{F}^{B} \tag{4.22}$$

where the global or system bending stiffness matrix, $\underline{\underline{K}}^{B}$, is constructed from the union of all the elemental bending stiffness matrices $\underline{\underline{k}}^{bi}$ and the global bending force vector, $\underline{F}^{b}$, is constructed from the union of all the elemental bending force vectors, $\underline{f}^{bi}$.

C. THE FINITE ELEMENT BAR EQUATION DEVELOPMENT

In a similar manner to the beam equation, the Galerkin Finite Element Method is applied to the bar equilibrium equation (1.2) to approximate the axial displacements at the endpoints of a bar element. However, the bar equation is only a second order linear differential equation. Therefore, a family of linear shape functions which posses the Kronecker Delta property, are used in order to maintain the necessary function continuity only. An approximate solution, $\tilde{u}$, for axial displacement, u, is formed as follows:

$$u \approx \tilde{u} = \underline{G}^{T}\underline{u} \tag{4.23}$$

where u is the exact solution of the bar equation in continuous space, $\tilde{u}$ is the approximate solution in discrete space, $\underline{G}^{T}$ is the transpose of a column vector of the linear shape functions, and $\underline{u}$ is the vector of axial displacements.

After the approximation is formulated, the next step in the Galerkin method is to form the residual, R, in the following format:

$$R=\mathfrak{L}(\tilde{u})+p_x(s) \qquad \textbf{(4.24)}$$

where p_x is the axial excitation force and $\mathfrak{L}$ denotes the differential operator which in the case of the bar equilibrium equation is defined by:

$$\mathfrak{L}(u)=[AE(u)']' \qquad \textbf{(4.25)}$$

With substitution, the residual becomes:

$$R=[AE(\underline{G}^T\underline{u})']'+p_x(s) \qquad \textbf{(4.26)}$$

From the residual, the Galerkin Equation is formed:

$$\int_D \underset{\sim}{G}(R)\,ds=\underline{0} \qquad \textbf{(4.27)}$$

where $\underline{0}$ represents the null vector. Further substitution into the residual equation results in :

$$\int_D \underline{G}[AE(\underline{G}^T\underline{u})']'ds+\int_D \underline{G}p_x(s)\,ds=\underline{0} \qquad \textbf{(4.28)}$$

Unlike the beam equation development, only single integration by parts is performed to solve the Galerkin Equation. This results in:

$$AE\underline{G}(\underline{G}^T\underline{u})'|_B-\int_D \underline{G}'[AE(\underline{G}^T\underline{u})']\,ds+\int_D \underline{G}p_x(s)=\underline{0} \qquad \textbf{(4.29)}$$

where $|_B$ represents evaluation at the boundaries of the structure. Recognizing that the axial displacement vector is constant, Equation (4.29) is rewritten as:

$$\underline{G}(AE\underline{G}^{T})'\underline{u}|_{B} - \int_{D} \underset{\sim}{G'}\,[AE(\underline{G}^{T'}]\,ds\underline{u} + \int_{D} \underline{G}p_{x}(s) = \underline{0} \qquad \textbf{(4.30)}$$

From the bar equilibrium Equation (1.2), the axial force, F, is defined by:

$$F = AEu' \qquad \textbf{(4.31)}$$

Thus, the boundary term load vectors are defined by:

$$\underline{P} = AE\underline{G}(\underline{G}^{T})'\underline{u}|_{B} \qquad \textbf{(4.32a)}$$

Additionally, for convenience a system stiffness Matrix, $\underline{\underline{K}}^{A}$, is defined by:

$$\underline{\underline{K}}^{A} = \int_{D} \underline{G'}\,[AE(\underline{G}^{T}]\,ds \qquad \textbf{(4.32b)}$$

and a system Force vector, $\underline{F}^{a}$, by:

$$\underline{F}^{a} = \int_{D} \underline{G}p_{x}(s) \qquad \textbf{(4.32c)}$$

Substitution of Equations (4.32 a through d) into equation (4.30) results in the following system of linear algebraic equations:

$$\underline{P} - \underline{\underline{K}}^{A}\underline{u} + \underline{F}^{a} = \underline{0} \qquad \textbf{(4.33)}$$

Further simplification is possible by defining $\underline{F}^{A}$ as the load vector of internal and external applied lateral loads by:

$$\underline{F}^{A} = \underline{F}^{a} + \underline{P} \qquad \textbf{(4.34)}$$

Thus, Equation (4.33) reduces to:

$$\underline{\underline{K}}^A \underline{u} = \underline{F}^A \tag{4.35}$$

here the global or system axial stiffness matrix, $\underline{\underline{K}}^A$, is constructed from the union of all the elemental axial stiffness matrices $\underline{\underline{k}}^{ai}$ and the global axial force vector, $\underline{F}^a$, is constructed from the union of all the elemental axial force vectors, $\underline{f}^{ai}$.

D. THE ELEMENTAL STIFFNESS MATRIX

The global Galerkin FEM Equations (4.22 and 4.35) are constructed from the union of elemental axial and bending stiffness matrices, $\underline{\underline{k}}^{ai}$ and $\underline{\underline{k}}^{bi}$ and axial and lateral force vectors, $\underline{\underline{f}}^{ai}$ and $\underline{\underline{f}}^{bi}$. For the beam element, the elemental degrees of freedom in which the elemental forces act are shown in Figure (4.2).

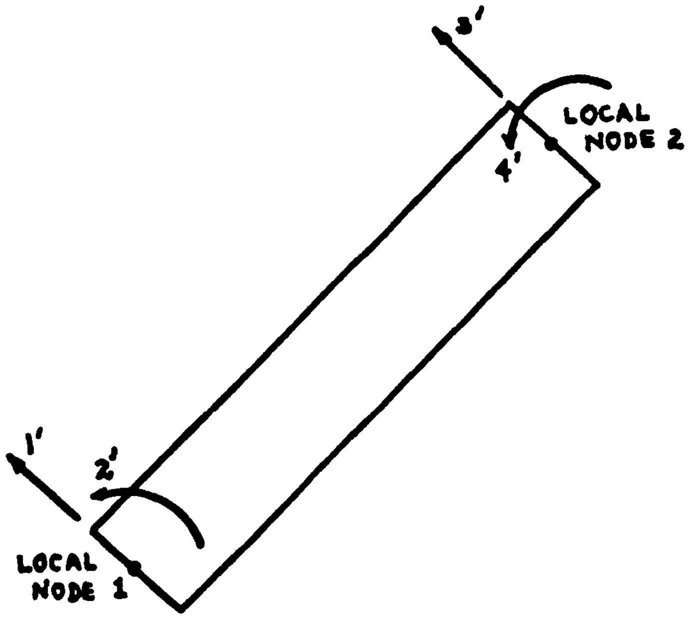


Figure 4.2 Beam Element - Degrees of Freedom

Thus, the stiffness matrix, $\underline{\underline{k}}^{bi}$ for bending results in a 4 X 4 matrix of the form:

$$\underline{\underline{k}}^{bi} = \begin{bmatrix} k_{11}^{bi} & k_{12}^{bi} & k_{13}^{bi} & k_{14}^{bi} \\ k_{21}^{bi} & k_{22}^{bi} & k_{23}^{bi} & k_{24}^{bi} \\ k_{31}^{bi} & k_{32}^{bi} & k_{33}^{bi} & k_{34}^{bi} \\ k_{41}^{bi} & k_{42}^{bi} & k_{43}^{bi} & k_{44}^{bi} \end{bmatrix} \tag{4.36}$$

For the bar element, the elemental degrees of freedom in which the elemental forces act are shown in Figure (4.3).

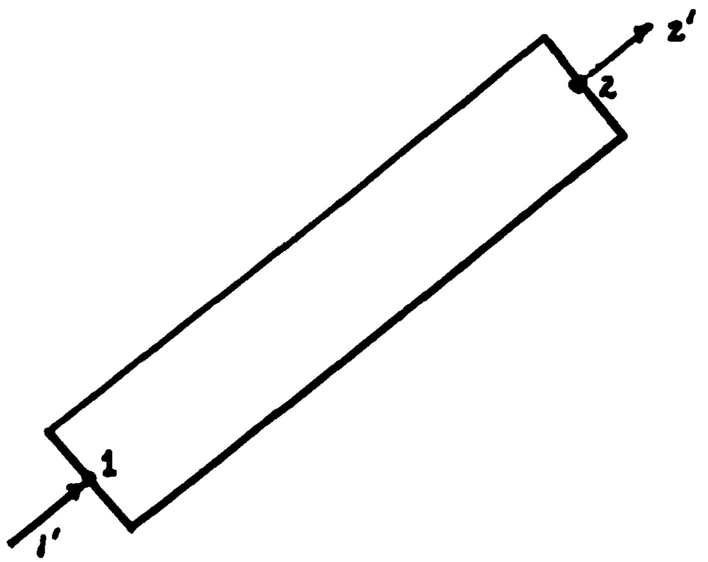


Figure 4.3 Bar Element - Degrees of Freedom

Thus the stiffness matrix, $\underline{\underline{k}}^{ai}$, for axial force results in a 2 X 2 matrix of the form:

$$\underline{k}^{ai} = \begin{bmatrix} k_{11}^{ai} & k_{12}^{ai} \\ k_{21}^{ai} & k_{22}^{ai} \end{bmatrix} \qquad (4.37)$$

To simplify, the elemental degrees of freedom are redefined for bar-beam elements as depicted in Figure (4.4).

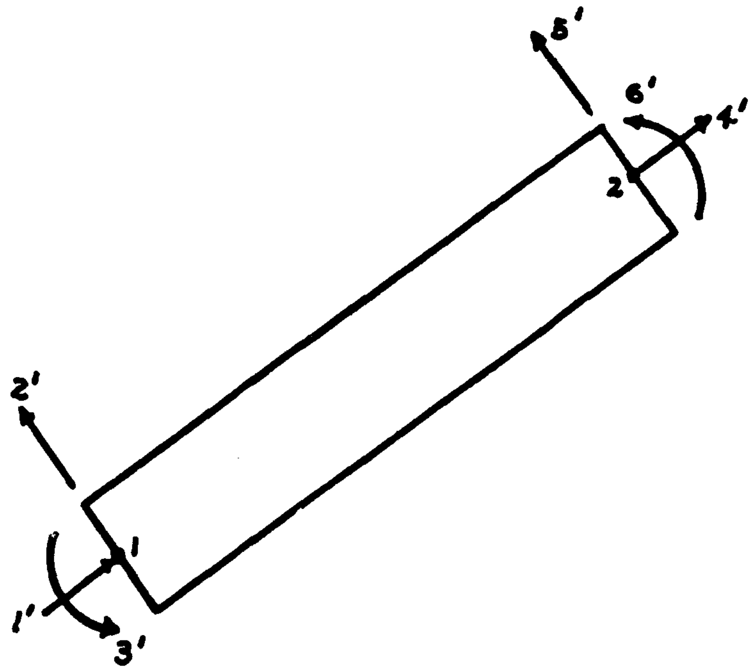


Figure 4.4 Bar-Beam Element - Degrees of Freedom

This results in a combined 6 X 6 stiffness matrix, $\underline{k}^{i}$, of the form:

$$
\underline{k}^{i} = \begin{bmatrix} k_{11}^{ai} & 0 & 0 & k_{12}^{ai} & 0 & 0 \\ 0 & k_{11}^{bi} & k_{12}^{bi} & 0 & k_{13}^{bi} & k_{14}^{bi} \\ 0 & k_{21}^{bi} & k_{22}^{bi} & 0 & k_{23}^{bi} & k_{24}^{bi} \\ k_{21}^{ai} & 0 & 0 & k_{22}^{ai} & 0 & 0 \\ 0 & k_{31}^{bi} & k_{32}^{bi} & 0 & k_{33}^{bi} & k_{34}^{bi} \\ 0 & k_{41}^{bi} & k_{42}^{bi} & 0 & k_{43}^{bi} & k_{44}^{bi} \end{bmatrix} \qquad \textbf{(4.38)}
$$

The elemental displacements and forces follow suit and are defined as follows:

The elemental displacements vector, $\underline{\delta}^{i'}$, becomes:

$$
(\underline{\delta}^{i'})^{T} = < \delta_1^{i'}, \delta_2^{i'}, \delta_3^{i'}, \delta_4^{i'}, \delta_5^{i'}, \delta_b^{i'} > \qquad \textbf{(4.39)}
$$

where for the ith element

$\delta_1^{i'}$	=	the axial displacement at local node 1
$\delta_2^{i'}$	=	the lateral displacement at local node 1
$\delta_3^{i'}$	=	the beam slope at local node 1
$\delta_4^{i'}$	=	the axial displacement at local node 2
$\delta_5^{i'}$	=	the lateral displacement at local node 2
$\delta_6^{i'}$	=	the beam slope at local node 2

The elemental force vector, $\underline{f}^{i'}$, becomes:

$$
(\underline{f}^{i'})^{T} = < f_1^{i'}, f_2^{i'}, f_3^{i'}, f_4^{i'}, f_5^{i'}, f_6^{i'} > \qquad \textbf{(4.40)}
$$

where for the ith element

$f_1^{i'}$	=	the axial force at local node 1
$f_2^{i'}$	=	the lateral force at local node 1
$f_3^{i'}$	=	the moment at local node 1
$f_4^{i'}$	=	the axial force at local node 2
$f_5^{i'}$	=	the lateral force at local node 2
$f_6^{i'}$	=	the moment at local node 2

Thus, the combination of the Galerkin Beam and Bar Equations for each element simplifies to:

$$\underline{\underline{k}}^{i'}\underline{\delta}^{i'}=\underline{f}^{i'} \tag{4.41}$$

where the elemental stiffness matrix, $\underline{\underline{k}}^{i'}$, in terms of known quantities becomes:

$$\underline{\underline{k}}^{i'}=\begin{bmatrix} AE/\ell_i & 0 & 0 & -AE/\ell_i & 0 & 0 \\ 0 & 12EI/\ell_i^3 & 6EI/\ell_i^2 & 0 & -12EI/\ell_i^3 & 6EI/\ell_i^2 \\ 0 & 6EI/\ell_i^2 & 4EI/\ell_i & 0 & -6EI/\ell_i^2 & 2EI/\ell_i \\ -AE/\ell_i & 0 & 0 & AE/\ell_i & 0 & 0 \\ 0 & -12EI/\ell_i^3 & -6EI/\ell_i^2 & 0 & 12EI/\ell_i^3 & -6EI/\ell_i^2 \\ 0 & 6EI/\ell_i^2 & 2EI/\ell_i & 0 & -6EI/\ell_i^2 & 4EI/\ell_i \end{bmatrix} \tag{4.42}$$

It should be noted that the bar and beam have uncoupled behavior.

E. COORDINATE TRANSFORMATION OF THE ELEMENTAL SYSTEM OF EQUATIONS

For curved structures such as the arch, each element has a unique orientation with respect to the global x and y axes. Therefore, to solve the global system of equations, the elemental Galerkin Equation (4.41) is transformed from local to global coordinates. The horizontal and vertical axes of the arch are chosen for a global reference coordinate system. Figure (4.5) depicts the angle the i^{th} element makes with the horizontal x-axis as α_i, and the compliment angle, β_i, as the angle the i^{th} element makes with the vertical y-axis.

From these definitions, the local displacements and forces, marked by a prime to indicate element degree of freedom are defined in terms of the reference coordinates axes as follows:

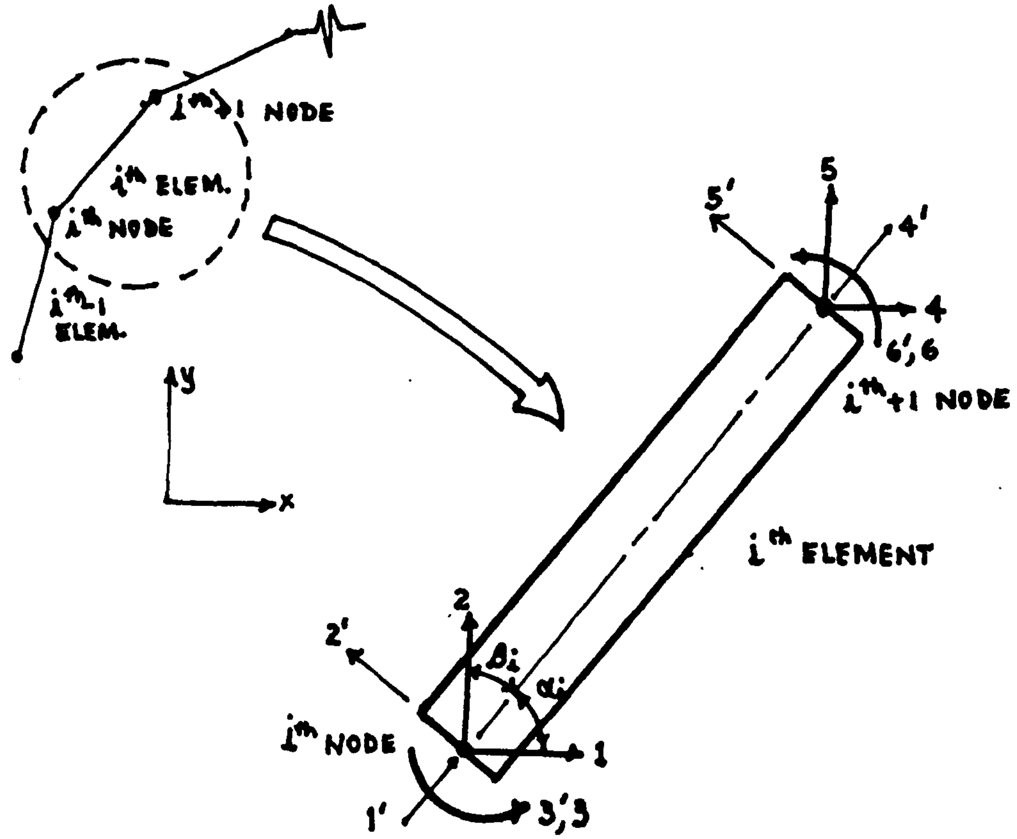


Figure 4.5 Bar-Beam Element Degrees of Freedom Transformation

$$\delta_1^{i'} = \delta_1^i \cos(\alpha_i) + \delta_2^i \cos(\beta_i)$$

$$\delta_2^{i'} = -\delta_1^i \cos(\beta_i) + \delta_2^i \cos(\alpha_i)$$

$$\delta_3^{i'} = \delta_3^i$$

$$\delta_4^{i'} = \delta_4^i \cos(\alpha_i) + \delta_5^i \cos(\beta_i)$$

$$\delta_5^{i'} = -\delta_4^i \cos(\beta_i) + \delta_5^i \cos(\alpha_i)$$

$$\delta_6^{i'} = \delta_6^i$$

and

$$f_1^{i'} = f_1^i \cos(\alpha_i) + f_2^i \cos(\beta_i) \qquad \textbf{(4.45)}$$

$$f_2^{i'} = -f_1^i \cos(\beta_i) + f_2^i \cos(\alpha_i)$$

$$f_3^{i'} = f_3^i$$

$$f_4^{i'} = f_4^i \cos(\alpha_i) + f_5^i \cos(\beta_i) \qquad \textbf{(4.46)}$$

$$f_5^{i'} = -f_4^i \cos(\beta_i) + f_5^i \cos(\alpha_i)$$

$$f_6^{i'} = f_6^i$$

Accordingly, a transformation matrix, $\underline{\underline{\Gamma}}^i$, for the ith element becomes:

$$\underline{\underline{\Gamma}}^i = \begin{bmatrix} \cos(\alpha_i) & \cos(\beta_i) & 0 & 0 & 0 & 0 \\ -\cos(\beta_i) & \cos(\alpha_i) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(\alpha_i) & \cos(\beta_i) & 0 \\ 0 & 0 & 0 & -\cos(\beta_i) & \cos(\alpha_i) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \textbf{(4.47)}$$

which reduces the notation of Equations (4.45) and (4.46) to:

$$\underline{\delta}^{i'} = \underline{\underline{\Gamma}}^i \underline{\delta}^i \qquad \textbf{(4.48)}$$

and

$$\underline{f}^{i} = \underline{\underline{\Gamma}}^{i} \underline{f}^{i} \quad \textbf{(4.49)}$$

where

$$(\underline{\delta}^{i})^{T} = < \delta_{1}^{i}, \delta_{2}^{i}, \delta_{3}^{i}, \delta_{4}^{i}, \delta_{5}^{i}, \delta_{6}^{i} > \quad \textbf{(4.50)}$$

$$(\underline{f}^{i})^{T} = < f_{1}^{i}, f_{2}^{i}, f_{3}^{i}, f_{4}^{i}, f_{5}^{i}, f_{6}^{i} > \quad \textbf{(4.51)}$$

Thus, the transformed elemental stiffness equation becomes:

$$\underline{\underline{k}}^{i\prime} \underline{\underline{\Gamma}}^{i} \underline{\delta}^{i} = \underline{\underline{\Gamma}}^{i} \underline{f}^{i} \quad \textbf{(4.51)}$$

by substituting Equations (4.48) and (4.49) into Equation (4.41).

By multiplying both sides of Equation (4.52) with the inverse of the transformation matrix, $\underline{\underline{\Gamma}}^{i}$, an orthogonal matrix (i.e., $\Gamma^{-1}=\Gamma^{T}$), yields:

$$(\underline{\underline{\Gamma}}^{i})^{T} \underline{\underline{k}}^{i\prime} (\underline{\underline{\Gamma}}^{i}) \underline{\delta}^{i} = \underline{f}^{i} \quad \textbf{(4.53)}$$

where the elemental stiffness matrix, $\underline{\underline{k}}^{i}$, in terms of the global x and y coordinates is defined by:

$$\underline{\underline{k}}^{i} = (\underline{\underline{\Gamma}}^{i})^{T} \underline{\underline{k}}^{i\prime} (\underline{\underline{\Gamma}}^{i}) \quad \textbf{(4.54)}$$

F. SOLUTION

Recall from the Beam and Bar FEM development that the global system of equations result from the union of the elemental stiffness matrices and force vectors such that:

$$\underline{\underline{K}}\,\underline{\Delta} = \underline{F} \qquad \textbf{(4.55)}$$

where the global or system force vector, $\underline{F}$, is the union of the transformed local force vectors, $\underline{f}^i$, and the global or system stiffness matrix, $\underline{\underline{K}}$, is the union of transformed local stiffness matrices, $\underline{\underline{k}}^i$. Thus, Equation (4.55) is solved for the global displacement vector, $\underline{\Delta}$.

These global horizontal, vertical, and rotational degrees of freedom are transformed back to local axial, lateral, and rotational displacements by the same transformation relationships of section E (Equations 4.45 and 4.46). From these local displacements, the virtual loads at the element endpoints are computed from Equation (4.41):

$$\underline{\underline{k}}^{i'}\underline{\delta}^{i'} = \underline{f}^{i'} \qquad \textbf{(4.56)}$$

where the elemental stiffness matrix, $\underline{\underline{k}}^{i'}$, is defined by Equation (4.42).

The node point virtual loads, $\underline{f}^{i'}$, equate to the virtual axial and lateral forces, and bending moments located at the endpoints of each element. From Equation (4.2) and (4.5), bending and axial stresses are calculated. For continuity, the stresses of internal global nodal points are averaged since physically, local nodal point 2 of the i^{th} element is the same point as local nodal point 1 of the i^{th} + 1 element. Therefore, using Equation (4.1), the normal stresses can be determined for each global nodal point.

V. PROGRAM DESCRIPTION AND VALIDATION

From the development of Chapters II and IV, a VAX Fortran 77 Code for FEM analysis of an arch was written to interface with the DOT software package. The main program, ARCH_OPT.FOR, and associated common program, ARCH_COM.FOR, are contained in Appendix C. Briefly, ARCH_OPT.FOR opens and reads an input file, ARCH_IN.DAT, before it is divided into several subroutines that perform the FEM analysis. Table 5.1 lists the input data fields required of ARCH_IN.DAT along with a brief description of each.

TABLE 5.1 ARCH_IN.DAT FIELD PARAMETERS

Input File form:	**ANGLE, RADIUS, YOUNG, YIELD,** **NEL, METHOD, IPRINT,** **DV1BG, DV1LO, DV1UP,** **DV2BG, DV2LO, DV2UP,** **CLAN, FX, FY, FM, FA,** **OPTDCS, ITERATE, PRCSN,** **BX1, BY1, BM1, BX2, BY2, BM2,** **LABEL**
Parameter	Description
ANGLE	The angle from 0 to 359 degrees subtended by the arch structure.
RADIUS	The length of the arch radius of curvature. (The dimension is arbitrary, however all remaining inputs must be consistent.)
YOUNG	Young' Modulus of Elasticity for the arch material.
YIELD	The yield strength of the arch material. If a factor of safety is desired, it should be accounted for prior to input.
NEL	An integer number of elements, from 1 to 32, used to approximate the arch structure.

METHOD The optimizer method to be used.

- **METHOD = 0 or 1**: Modified Method of Feasible Directions
- **METHOD = 2**: Sequential Linear Programming

IPRINT On screen print control parameter. Integers from 0 to 5 indicate increasing screen printout.

DV_BG The best guess for design variable 1, the base dimension, or 2, the height dimension. Nodal point dimensions are initialized to the best guess value, thus establishes the optimization starting point.

DV_LO The lower limit or side constraint for design variable 1, the base dimension, or 2, the height dimension.

DV_UP The upper limit or side constraint for design variable 1, the base dimension, or 2, the height dimension.

CLAN An integer from 1 to NEL + 1 that indicates the node at which the concentrated load is to be applied.

FX The magnitude of the concentrated load in the horizontal direction applied at node CLAN.

FY The magnitude of the concentrated load in the vertical direction applied at node CLAN.

FM The magnitude of the concentrated moment applied at node CLAN.

FA The magnitude of the uniformly distributed load in the radial direction which spans the entire length of the arch.

OPTDCS Optimization option

- **OPTDCS = 1**: Optimize the dimensions of the problem.
- **OPTDCS = 2**: Do not optimize the problem. Based on the initial design, calculate the stress distribution only.

ITERATE The number of iterations performed. The resulting optimized variables are re-entered into DOT and the optimization performed ITERATE times to effect an iteration.

PRCSN Computer precision used by the equation solver.

- **PRCSN = 1**: single precision
- **PRCSN = 2**: double precision

BX_		Boundary conditions for horizontal displacement at 1, the first node of the arch, node 1, or 2, the last node of the arch. node NEL + 1.
	BX_ = 0:	The node is free to move horizontally.
	BX_ = 1:	The node is not free to move horizontally.
BY_		Boundary conditions for vertical displacement at 1, the first node of the arch, node 1, or 2, the last node of the arch, node NEL + 1.
	BY_ = 0:	The node is free to move vertically.
	BY_ = 1:	The node is not free to move vertically.
BM_		Boundary conditions for the beam slope at 1, the first node of the arch, node 1, or 2, the last node of the arch, node NEL + 1.
	BM_ = 0:	The node is free to rotate.
	BM_ = 1:	The node is not free to rotate.
LABEL		A character string used to identify the output.

As outlined in Figure 5.1, the main program, ARCH_OPT.FOR is divided into subroutines. In general, subroutine Geometry is called in order to generate the x and y coordinates of the global nodal points and determine the orientation of each element. Following Geometry, subroutine Optimization_tool establishes the DOT parameters prior to the first call of the DOT program. The first call serves only to record the DOT parameters selected in DOT's internal arrays. After DOT is called, the Optimization_tool subroutine, calls Eval to evaluate the objective function and constraint functions originally outlined in the problem formulation of Chapter II.

As detailed in Chapter IV, the constraint functions are made functions of the design variables through Finite Element

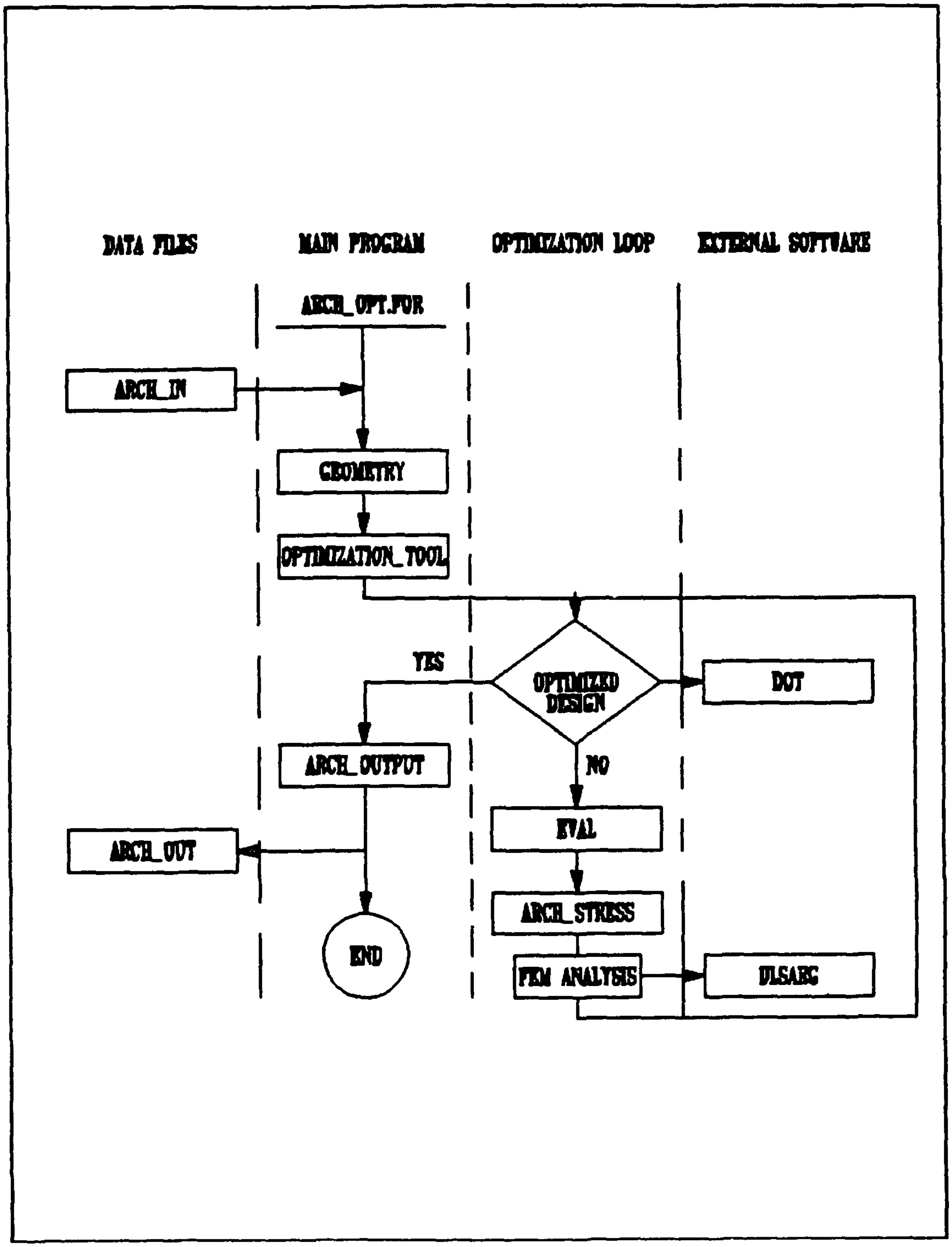


Figure 5.1 Arch Opt Program Structure

Method analysis. Subroutines Form and Force_vector develop the global stiffness matrix and force vector, which are modified by subroutine Bndary for the appropriate boundary conditions. The equation solver, L2ARG, from the IMSL library is called to solve for the global displacements, which in turn are used to calculate the nodal stresses. Once the constraints are evaluated for the initial design, the problem is returned to DOT where the move direction is computed and an updated design point chosen. The objective and constraint functions are reevaluated for the updated design point before returning to DOT for further iteration.

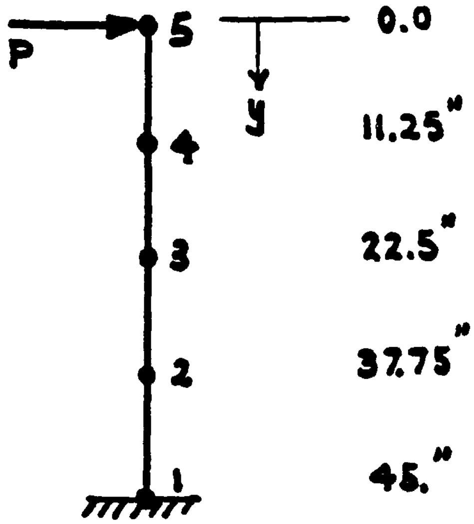
Once termination criteria for optimization are reached, the main program creates the output file, ARCH_OUT.DAT. This file contains the problem parameters, optimized design variables, and the resulting objective function value along with a variety of additional information. Summarizing, for a given geometry, loading, and set of end conditions, the program is capable of finding the optimum cross section dimensions of each nodal point along the length of the arch.

To validate the FEM analysis, several non-optimum straight beam and arch problems with known analytical solutions were solved. A straight cantilever structure, subject to a concentrated lateral end load, axial load, and end moment; and a quarter cantilever arch, subject to a lateral end load were analyzed. These test problems established the program error for stress and displacement calculations. Additionally, the

quarter cantilever arch and a hinged-hinged semi-circular arch structure, subject to a lateral load on the axis of symmetry, establish trends in a relationship between the number of elements used to approximate the arch and accuracy. The remainder of this chapter is a summary of the results and the conclusions drawn from each validation problem studied. The complete solution of each problem is contained in Appendix D.

A. VALIDATION I: CANTILEVER BEAM

A cantilever beam is subject to a concentrated end load as shown in Figure 5.2. [Ref. 8]



$$L = 45.00\ inches$$
$$B = 1.50\ inches$$
$$H = 3.00\ inches$$
$$I = \frac{BH^3}{12} = 3.375\ inches^4$$
$$P = 1{,}000.0\ lbf$$
$$M = P \cdot y$$
$$\sigma_3 = \frac{M\frac{h}{2}}{I}$$

Figure 5.2 Validation Case #1

ARCH_OPT.FOR was run for this beam structure using an angle of 45.0 X 10^{-6} radians and a radius of 10^{6} inches to

approximate a straight beam of 45 inches. The four element FEM solution is compared to the analytical solution in Table 5.2.

TABLE 5.2

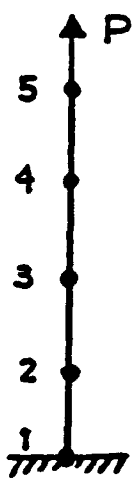
NODE	THEORETICAL STRESS	FEM ANALYTICAL STRESS	% ERROR
1	20000.0	19999.7	0.0015
2	15000.0	14999.7	0.0020
3	10000.0	9999.8	0.0020
4	5000.0	4999.9	0.0020
5	0.0	0	0.0000

where the percent error is defined as:

*% Error = (Theory - FEM Analysis)/ Theory * 100*

B. VALIDATION II: PRISMATIC BAR

Similarly, a prismatic bar is subject to an axial load as shown in Figure 5.3. [Ref. 8]



$L = 45.00\ inches$

$B = 1.50\ inches$

$H = 3.00\ inches$

$A = B \cdot H = 4.50\ inches^2$

$P = 1,000.0\ lbf$

$\sigma_n = \frac{P}{A} = 222.2$

Figure 5.3 Validation Case #2

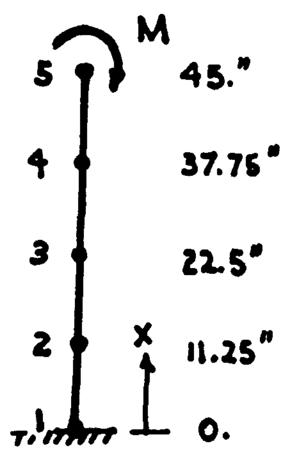
Input values for angle and radius remained the same to approximate the straight bar. The four element FEM solution is compared in Table 5.3.

TABLE 5.3

NODE	THEORETICAL STRESS	FEM ANALYTICAL STRESS	% ERROR
1	222.2	222.2	0.0000
2	222.2	222.2	0.0000
3	222.2	222.2	0.0000
4	222.2	222.2	0.0000
5	222.2	222.2	0.0000

C. VALIDATION III: CANTILEVER BEAM

The cantilever beam is subject to a concentrated moment at the free end as shown in Figure 5.4. [Ref. 8]



$L = 45.00\ inches$

$B = 1.50\ inches$

$H = 3.00\ inches$

$$I = \frac{BH^3}{12} = 3.375\ inches^4$$

$E = 30 \times 10^6\ psi$

$M = 10{,}000\ lbf$

$$Slope\ S' = \frac{Mx}{EI}$$

$$Displacement\ S = \frac{Mx^2}{2EI}$$

Figure 5.4 Validation Case #3

The four element FEM solution for both slope and displacement is compared in Table 5.4.

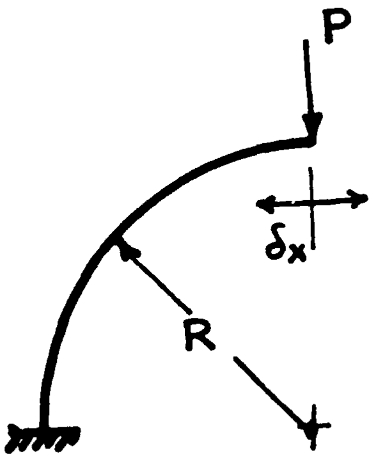
TABLE 5.4

NODE	THEORETICAL SLOPE	FEM ANALYTICAL SLOPE	% ERROR
1	0.00000000	0.00000000	0.0000
2	0.00111111	0.00111109	0.0019
3	0.00222222	0.00222218	0.0019
4	0.00333333	0.00333328	0.0016
5	0.00444444	0.00444438	0.0014

NODE	THEORETICAL DISPLACEMENT	FEM ANALYTICAL DISPLACEMENT	% ERROR
1	0.00000000	0.00000000	0.0000
2	0.00625000	0.00624985	0.0024
3	0.02500000	0.02499940	0.0024
4	0.05625000	0.05624880	0.0021
5	0.10000000	0.09999790	0.0021

D. DATION IV: CANTILEVER QUARTER ARCH

A cantilever quarter arch is subject to a lateral load as shown in Figure 5.5. [Ref. 8]



$L = 45.00\ inches$

$B = 1.50\ inches$

$H = 3.00\ inches$

$I = \frac{BH^3}{12} = 3.375\ inches^4$

$E = 30 \times 10^6\ psi$

$P = 1{,}000.0\ lbf$

$\delta_x = \frac{PR^3}{2EI}$

Figure 5.5 Validation Case #4

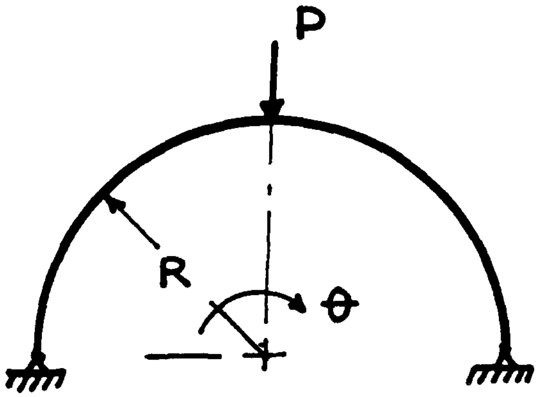
ARCH_OPT.FOR was run for this structure using an angle of 90.0 degrees and a radius of 45 inches. To approximate the arch a four, six, eight, ten and 12 element FEM solution is solved and compared to the analytical solution presented in Table 5.5.

TABLE 5.5

NODE	THEORETICAL δ_x	FEM ANALYTICAL δ_x	% ERROR
4	0.450000	0.446951	0.677556
6	0.450000	0.448382	0.359556
8	0.450000	0.448854	0.254667
10	0.450000	0.448790	0.268889
12	0.450000	0.449100	0.200000

E. VALIDATION V: HINGED-HINGED SEMI-CIRCULAR ARCH

A hinged-hinged semi-circular arch structure is subject to a lateral load along the axis of symmetry as shown in Figure 5.6. [Ref. 8]



$R = 32.0\ inches$

$B = 1.50\ inches$

$H = 3.00\ inches$

$P = 10{,}000.0\ lbf$

$$M = \frac{PR}{2}(1 - \cos\theta) - \frac{PR}{\pi}(8m\theta)$$

Figure 5.6 Validation Case #5

Results are tabulated in Table 5.6 for comparison of the four, six, eight, ten, 12, 14, and 16 element FEM solutions to the analytical solution. It should be noted that by using symmetry, the arch structure is approximated by twice the number of elements shown in the calculations.

TABLE 5.6

NODE	θ	THEORETICAL STRESS	FEM ANALYTICAL STRESS	% ERROR
4	0	0.0	0	0.000000
	22.5	11911.4	12338	3.581870
	45	11183.3	11971.7	7.049945
	67.5	2073.4	1043.2	49.685474
	90	25840.4	24725.3	4.315231
6	0	0.0	0	0.000000
	15	9293.9	9411.2	1.262367
	30	13108.3	13334.9	1.728775
	45	11183.3	11503.9	2.866917
	60	3650.1	4042.8	10.759982
	75	8978.0	8539.9	4.879970
	90	25840.4	25386.8	1.755283
8	0	0.0	0	0.000000
	11.25	7465.5	7509.3	0.586658
	22.5	11911.4	11997.3	0.721573
	33.75	13166.7	13291.6	0.948666
	45	11183.3	11342.3	1.421903
	56.25	6037.3	6224.5	3.099877
	67.5	2073.4	1865.4	10.029988
	78.75	12837.1	12616.3	1.720372
	90	25840.4	25615.1	0.87178
10	0	0.0	0	0.000000
	9	6206.4	6225.1	0.301171
	18	10509.0	10546.3	0.352057
	27	12801.8	12856.2	0.424639
	36	13028.5	13098.8	0.539901
	45	11183.3	11268.0	0.757518
	54	7311.7	7408.7	1.325965
	63	1509.2	1616.0	7.077882
	72	6081.5	5967.4	1.876789
	81	15273.5	15155.0	0.775860
	90	25840.4	25720.4	0.464280

NODE	θ	THEORETICAL STRESS	FEM ANALYTICAL STRESS	% ERROR
12	0	.0	0	0.000000
	15	9293.9	9309.8	0.171326
	30	13108.3	13139.3	0.236589
	45	11183.3	11227.5	0.395371
	60	3650.1	3704.7	1.497107
	75	8978.0	8917.1	0.678600
	90	25840.4	25777.3	0.244082
14	0	0.0	0	0.000000
	6.428571	4621.6	4624.5	0.063120
	12.85714	8290.8	8296.5	0.068954
	19.28571	10961.5	10970.1	0.078835
	25.71429	12600.0	12611.5	0.091081
	32.14286	13185.9	13200.1	0.107896
	38.557143	12711.6	12728.6	0.133432
	45	11183.3	11202.7	0.173611
	51.42857	8620.0	8641.9	0.253703
	57.85714	5054.1	5078.2	0.476611
	64.28571	530.4	556.4	4.907803
	70.71429	4894.3	4866.7	0.563998
	77.14286	11151.7	111123.1	0.256399
	83.57143	18163.1	18133.9	0.160800
	90	25840.4	25810.9	0.114053
16	0	0.0	0	0.000000
	11.25	7465.5	7465	0.006738
	22.5	11911.4	11911.1	0.002106
	33.75	13166.7	13167.7	0.007655
	45	11183.3	11186	0.024281
	56.25	6037.3	6042.2	0.080341
	67.5	2073.4	2066.5	0.330744
	78.78	12837.1	12828.8	0.065020
	90	25840.4	25831.4	0.034720

F. CONCLUSIONS

The four element approximation for a straight cantilever structure produced an error no greater than 0.016%. The cantilever quarter arch produced an error less than 0.70% for the four element model which reduced to less than 0.20% with 12 elements. The results of the hinged-hinged arch indicate, as expected, that the more elements used the better the solution. Considering only meaningful stresses, stresses in excess of 10,000 psi, the error is less than 2% for eight elements and less than 0.8% for 12 elements.

In general, the percent error recorded for the first three validation cases seemed insignificant. Four element approximations sufficed to solve the stresses, slopes, and displacements for straight structures. Therefore, it was concluded that the program was producing accurate results for analysis of straight beams.

Unfortunately, for the arch structures, the error of the four element model was significant (greater than 45%). However, the error reduced significantly when more elements were used to approximate the structure. Grid independence, (2% error), was not achieved for the hinged-hinged arch until at least eight elements are used to approximate the structure. This indicates that an element cannot be used to span more than 11.25 degrees of arch. The resulting trend, as expected, confirms that the more elements used, the better the model. However, computer time and computer error increase with

increase in the number of elements and models of more than eight elements were not used.

VI. CASE STUDIES

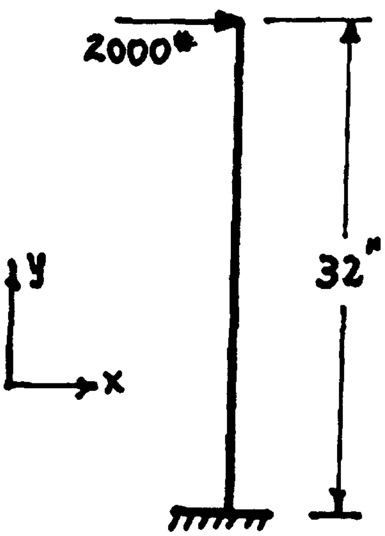
Results are presented for a number of cases with regard to optimization scheme and stress analysis. The case studies range from the simple cantilever beam to complex arch structures. In addition, for many cases one parameter of the same structure was modified and the problem was reoptimized to establish a comparison. The straight beam is examined first, followed by five cases studying the quarter cantilever arch with varied loadings. Cases #7 and #8 are symmetric semicircular arches comparing simply supported arch structures with fixed-end arch structures. The remaining cases are asymmetric semicircular arch structures. Cases #9 through #11 investigate various end conditions and Cases #12 through #14 various combined loadings. The cases conclude with Case #15 which combines a concentrated lateral load, applied moment, and distributed load across the arch structure.

For each case, interpretations of the results are accompanied by a schematic drawing of the structure modeled, a plot of the cross section dimensions and area as functions of nodal points, and a plot of the axial and bending stresses

as functions of nodal points. Eight elements were selected to model the arch structures and the material properties were selected such that the yield strength was imputed as 52,000 psi and Young's Modulus as 30,000,000 psi. For reference, the Modified Method of Feasible Directions will be referred to as Method 1 and the Sequential Linear Programming Method will be referred to as Method 2. Additionally, each endpoint, unless geometrically restricted by imposed boundary conditions, can have three 'means of displacement,' MOD. An endpoint can rotate about the z-axis, displace in the x direction, and displace in the y direction. For reference, an endpoint will be described by a number from zero to three reflecting the means of diplacement. As an example, a fixed end is considered to have zero means of displacement because it cannot rotate or displace in either the x or y direction. A free end which can rotate and displace in both the x and y direction is considered to have three means of displacement. A hinge which can only rotate has one MOD. The complete computer data printout is presented in Appendix D.

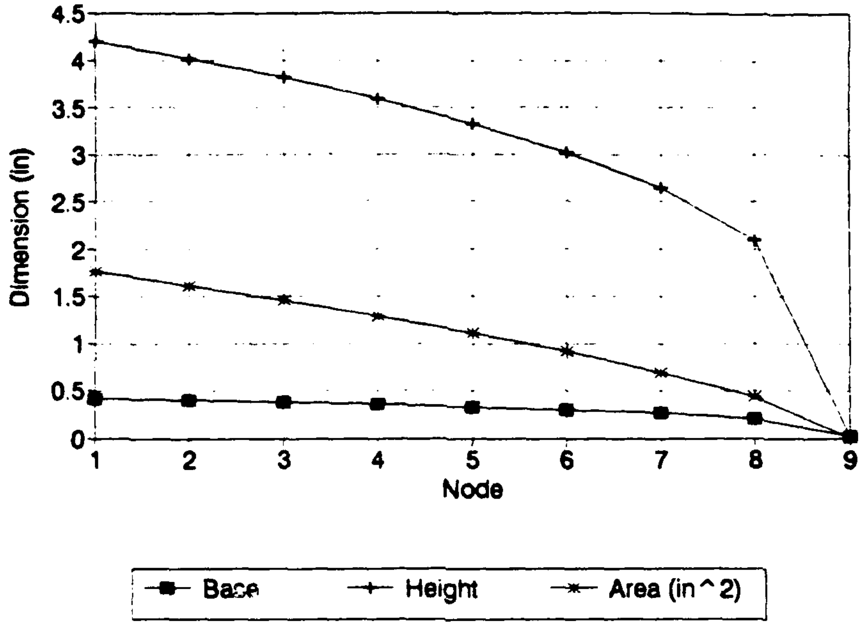
A. CASE #1: CANTILEVER BEAM WITH LATERAL LOADING

The cantilever beam was optimized first in order to provide guidance for adjusting the various parameters discussed in Chapter III. Satisfactory results were produced by turning the auto scaling function off, reducing CT and CTMIN, and establishing the termination criteria. Using the Modified Method of Feasible Directions, henceforth referred to as Method 1, the cross section dimensions and stresses were plotted. As expected, the dimensions form a parabolic function over the length of the beam. Furthermore, the beam exhibits only stress due to bending moments. The normal stresses are virtually nonexistent which likewise is as expected.

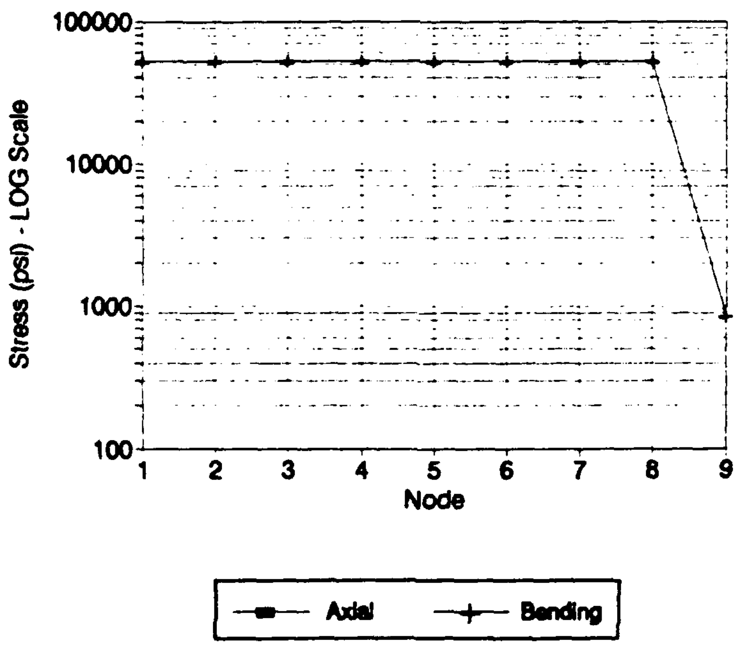


Loads		
Lateral	=	2,000 lbs
Axial	=	0 lbs
Moment	=	0 in-lbs
End conditions		
Node 1		0 MOD
Node 9		3 MOD
Dimensions		
Radius	=	not applicable
Theta	=	not applicable
Total volume		
Volume	=	33.13 in^3

Case #1



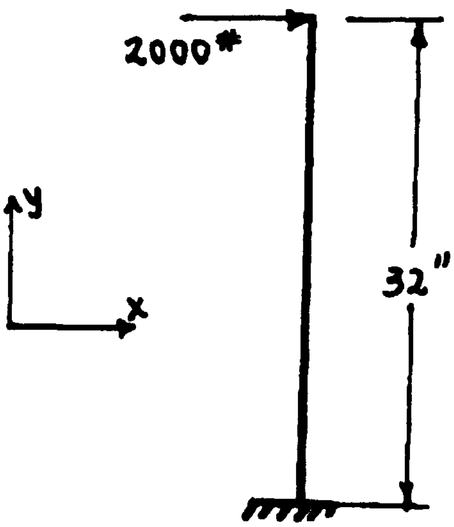
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
Dimension (in)
1 2 3 4 5 6 7 8 9
Node
Base
Height
Area (in^2)



100000
10000
1000
100
Stress (psi) - LOG Scale
1 2 3 4 5 6 7 8 9
Node
Axial
Bending

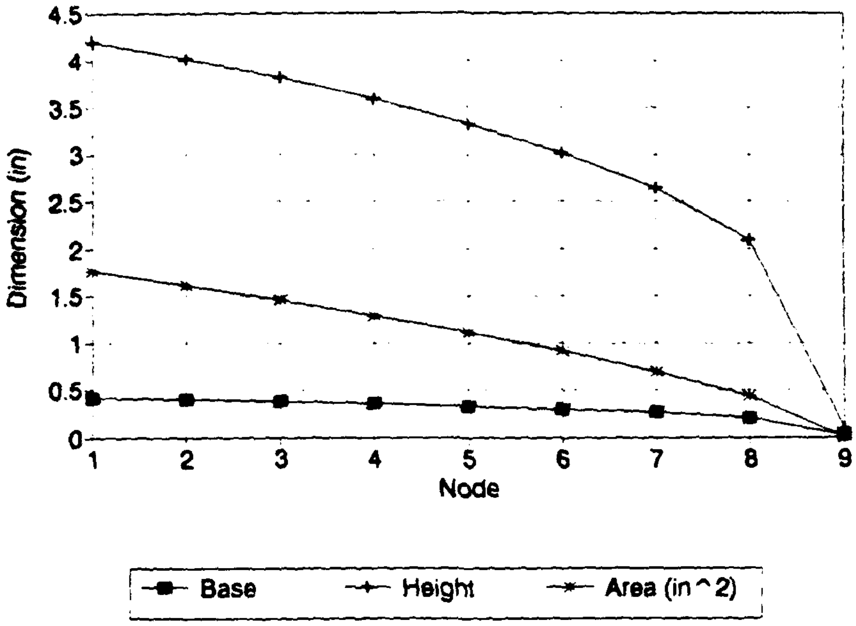
B. CASE #1A: CANTILEVER BEAM WITH LATERAL LOADING

For comparison with Case #1, the same cantilever beam was optimized using the Sequential Linear Programming Method, henceforth referred to as Method 2. The results are quite similar. In total structure volume, the difference is less than 0.07%. The only significant difference appears at nodal point 9, the free end. In theory, the free end of a beam can support no bending stresses. For this case, nodal point 9 has no stresses unlike the previous case which had relatively small bending stresses at nodal point 9. However, from this result alone it is not conclusive that Method 2 is superior to Method 1.

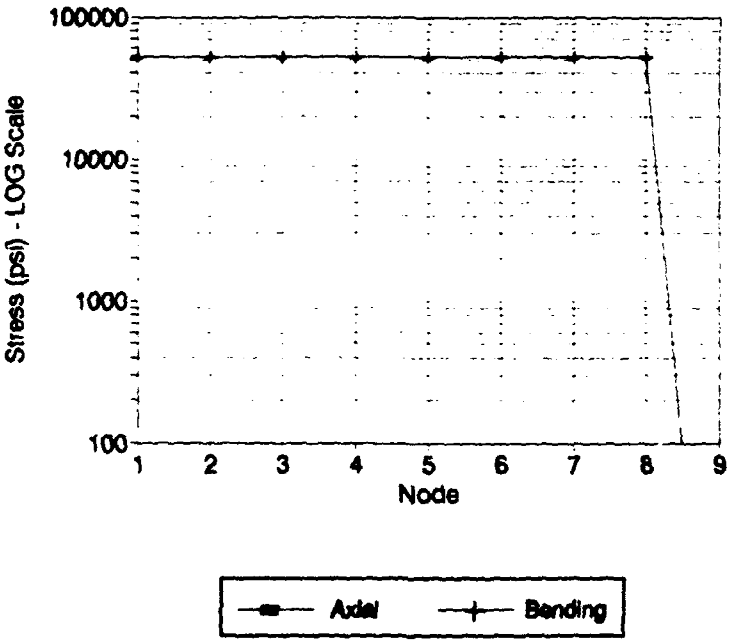


Loads		
Lateral	=	2,000 lbs
Axial	=	0 lbs
Moment	=	0 in-lbs
End conditions		
Node 1		0 MOD
Node 9		3 MOD
Dimensions		
Radius	=	not applicable
Theta	=	not applicable
Total volume		
Volume	=	33.15 in^3

Case #1a



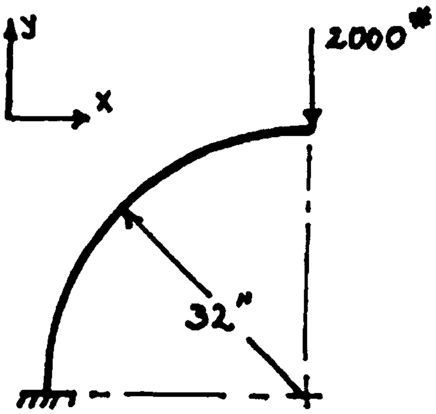
Dimension (in)
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
1 2 3 4 5 6 7 8 9
Node
Base
Height
Area (in^2)



Stress (psi) - LOG Scale
100000
10000
1000
100
1 2 3 4 5 6 7 8 9
Node
Axial
Bending

C. CASE #2: CANTILEVER ARCH WITH LATERAL LOADING

Since Case #1 did not strongly suggest a preferential method, the cantilever arch was optimized with Method 1. At most nodal points, the total stresses were well below the yield stress which indicates that this design is far from an optimum structure. Additionally, the height and base dimensions hovered around the initial starting point of 2 inches by 2 inches and produced a structure only 7.42% less in volume than that of the initial structure. It appears that the optimizer failed to achieve an optimum solution using this Method.



Loads

Lateral	=	2,000 lbs
Axial	=	0 lbs
Moment	=	0 in-lbs

End conditions

Node 1	0 MOD
Node 9	1 MOD

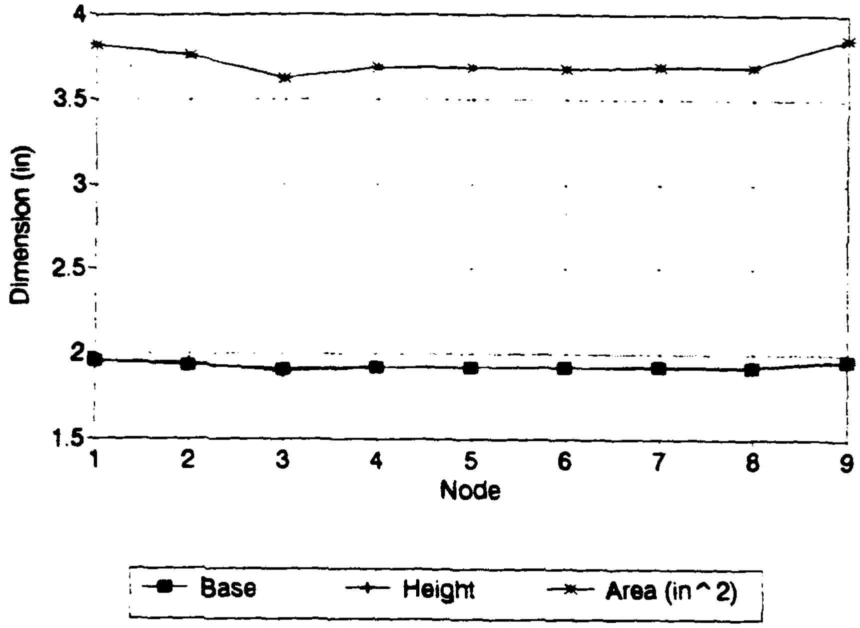
Dimensions

Radius	=	32 in
Theta	=	90 degrees

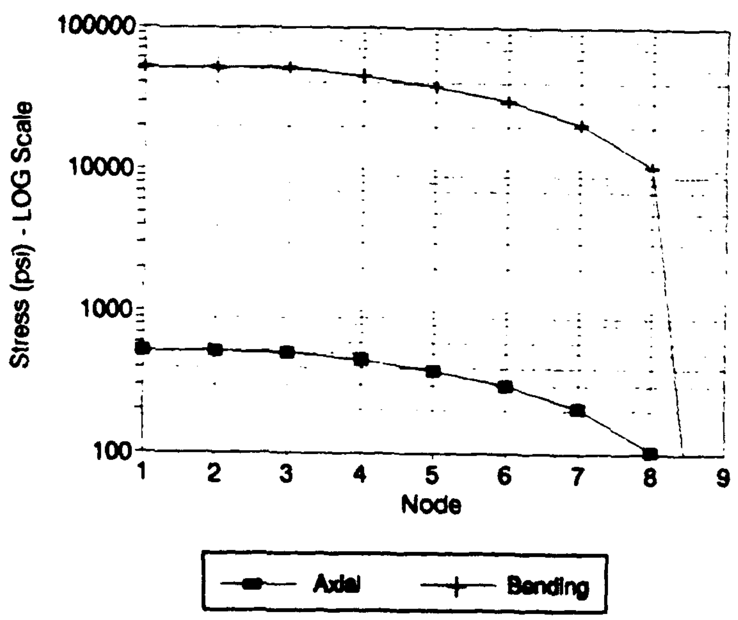
Total volume

Volume	=	186.15 in^3

Case #2



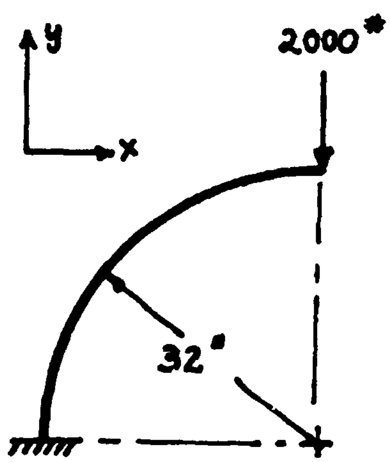
Dimension (in)
4
3.5
3
2.5
2
1.5
1 2 3 4 5 6 7 8 9
Node
Base
Height
Area (in^2)



Stress (psi) - LOG Scale
100000
10000
1000
100
1 2 3 4 5 6 7 8 9
Node
Axial
Bending

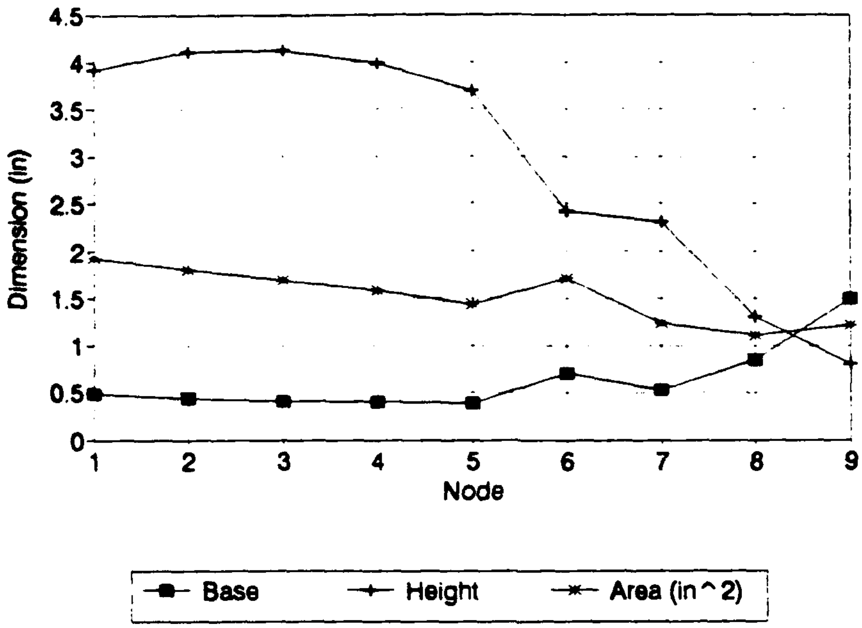
D. CASE #2A: CANTILEVER ARCH WITH LATERAL LOADING

For comparison, the same arch structure was reoptimized with Method 2. Each element of the structure now supports stresses equal to the yield stress producing an efficient structure. The total volume was reduced from the initial starting point by 61.32%. For this structure, Method 2 also produced results with fewer iterations than Method 1. With these observations in mind, Method 2 was selected as the preferred method for quarter arches. Additionally, it is interesting to note that the axial stresses only remotely effect the stress total for the first 5 nodal points, hence the first 45 degrees of arch. After node 5, the height reduces significantly, however the area remains roughly the same.

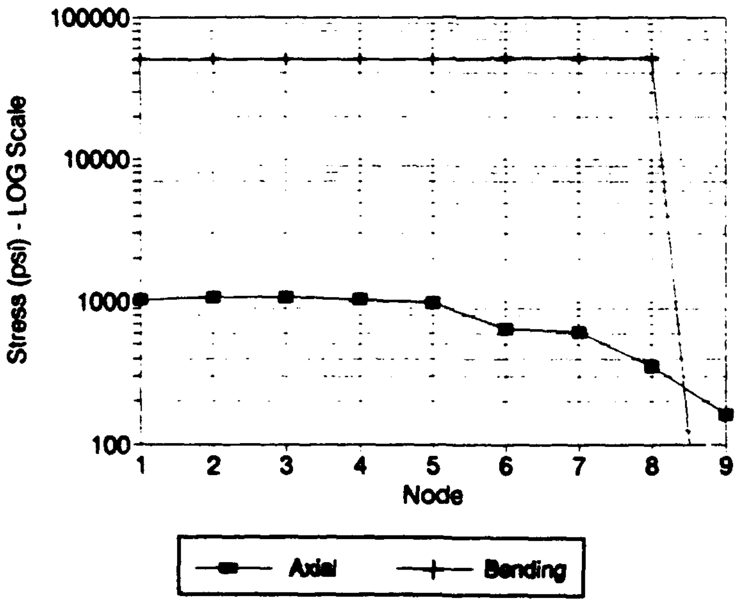


Loads		
Lateral	=	2,000 lbs
Axial	=	0 lbs
Moment	=	0 in-lbs
End conditions		
Node 1		0 MOD
Node 9		3 MOD
Dimensions		
Radius	=	32 in
Theta	=	90 degrees
Total volume		
Volume	=	77.78 in^3

Case #2a



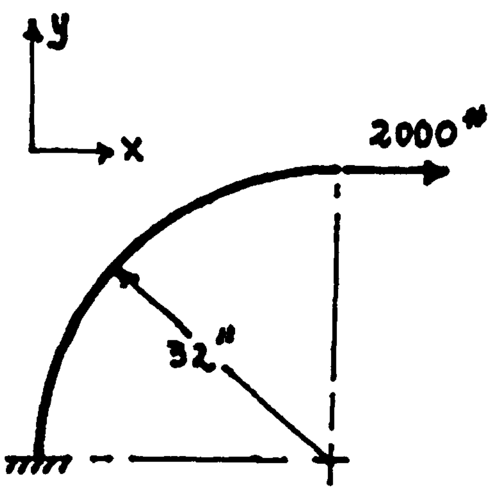
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
Dimension (in)
1
2
3
4
5
6
7
8
9
Node
Base
Height
Area (in^2)



100000
10000
1000
100
Stress (psi) - LOG Scale
1
2
3
4
5
6
7
8
9
Node
Axial
Bending

E. CASE #3: CANTILEVER ARCH WITH AXIAL LOADING

This case presents a quarter arch structure subject to an axial load vice the lateral load of Case #2. Unlike Case #2 the axial stresses increase significantly along the length of the arch and the bending stresses decrease. The net result is an arch structure of 27.15% less material. This seems to indicate a dominant relationship between area and bending stress. Additionally, this case exemplifies the difficulty experienced by approximating an arch of 90 degrees with eight straight segments. The plots appear very disjointed, hence the data points seem circumspect. However, the effect can be minimized as presented in Case #3a.



Loads

Lateral	=	0 lbs
Axial	=	2,000 lbs
Moment	=	0 in-lbs

End conditions

Node 1	0 MOD
Node 9	3 MOD

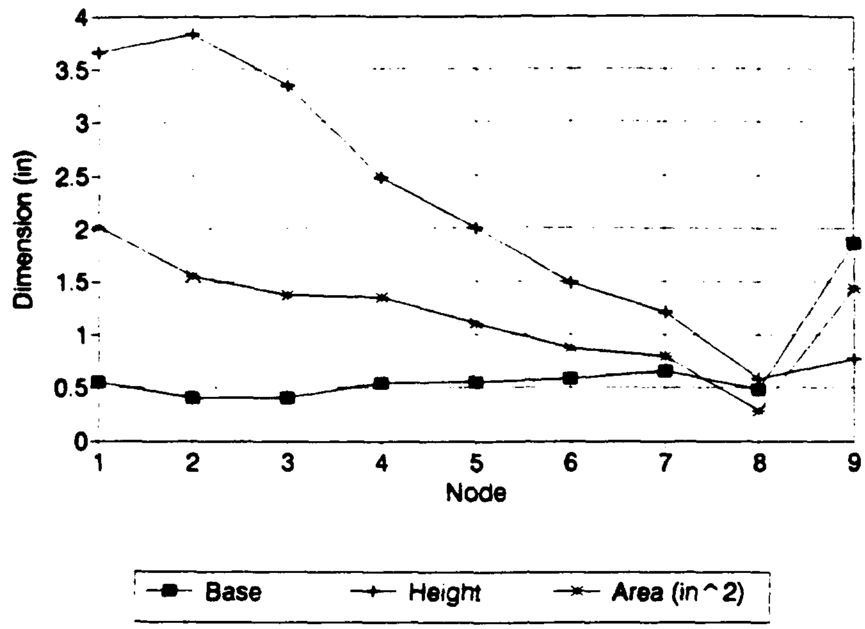
Dimensions

Radius	=	32 in
Theta	=	90 degrees

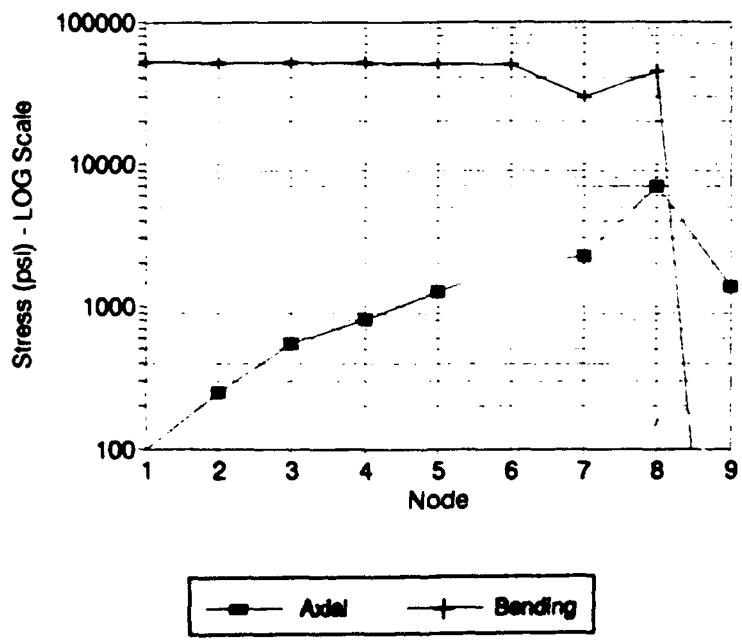
Total volume

Volume	=	56.66 in^3

Case #3



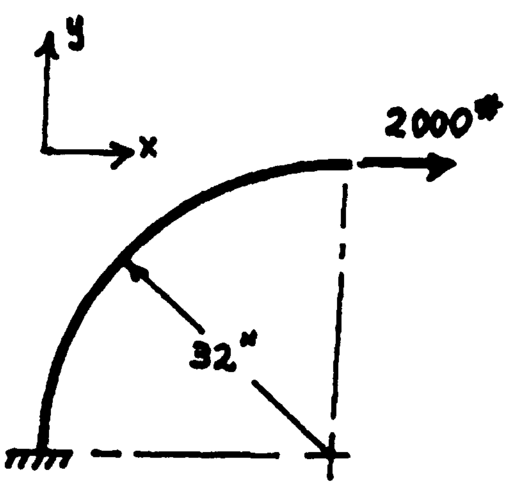
Dimension (in)
4
3.5
3
2.5
2
1.5
1
0.5
0
1 2 3 4 5 6 7 8 9
Node
Base
Height
Area (in^2)



Stress (psi) - LOG Scale
100000
10000
1000
100
1 2 3 4 5 6 7 8 9
Node
Axial
Bending

F. CASE #3A: CANTILEVER ARCH WITH LATERAL LOADING

Thus far, each case has started with an initial design of 2 inches by 2 inches. For comparison, the arch structure of Case #3 was optimized a second timeusing the results of Case #3 as the initial design. Reoptimizing had the desired effect of smoothing the results and in graphical form, both the area and stress curves take on a fairer shape. In terms of total structure volume, the reoptimized arch was 27.03% smaller than that of Case #3. In all subsequent cases this strategy of reoptimization will be referred to as a two-stage optimization strategy.



Loads

Lateral	=	0 lbs
Axial	=	2,000 lbs
Moment	=	0 in-lbs

End conditions

Node 1	0 MOD
Node 9	3 MOD

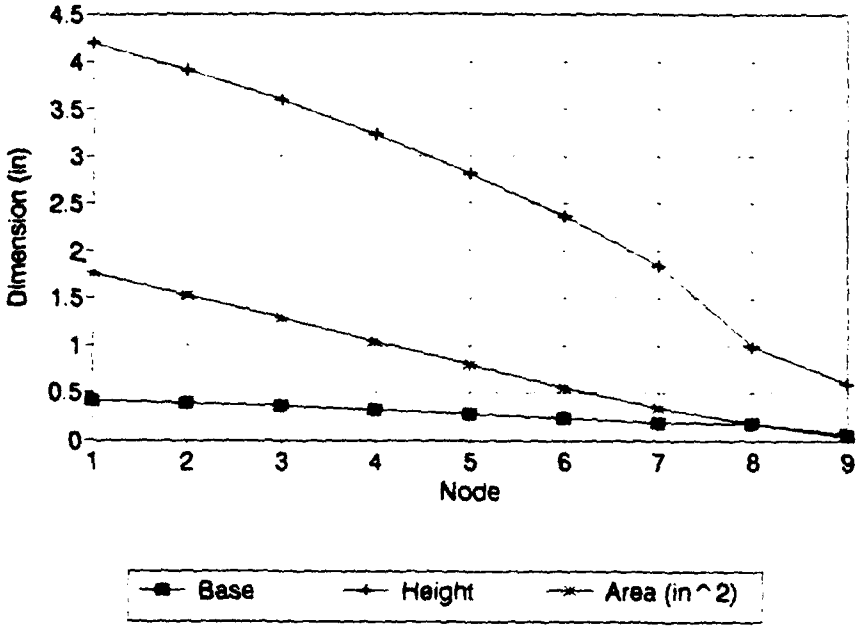
Dimensions

Radius	=	32 in
Theta	=	90 degrees

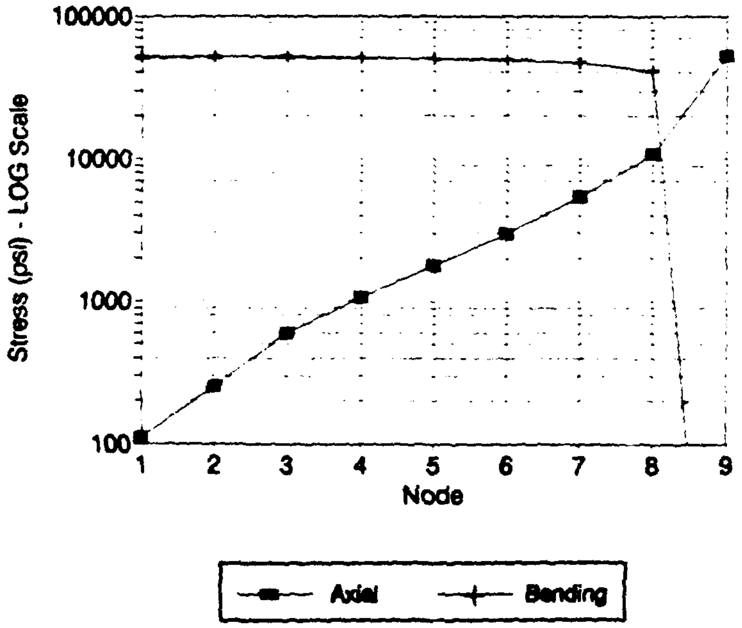
Total volume

Volume	=	41.34 in^3

Case #3a



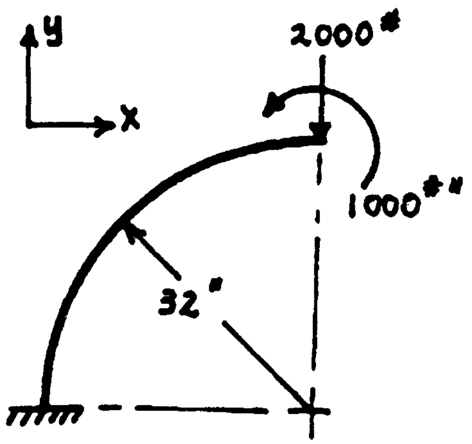
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
Dimension (in)
1 2 3 4 5 6 7 8 9
Node
Base
Height
Area (in^2)



100000
10000
1000
100
Stress (psi) - LOG Scale
1 2 3 4 5 6 7 8 9
Node
Axial
Bending

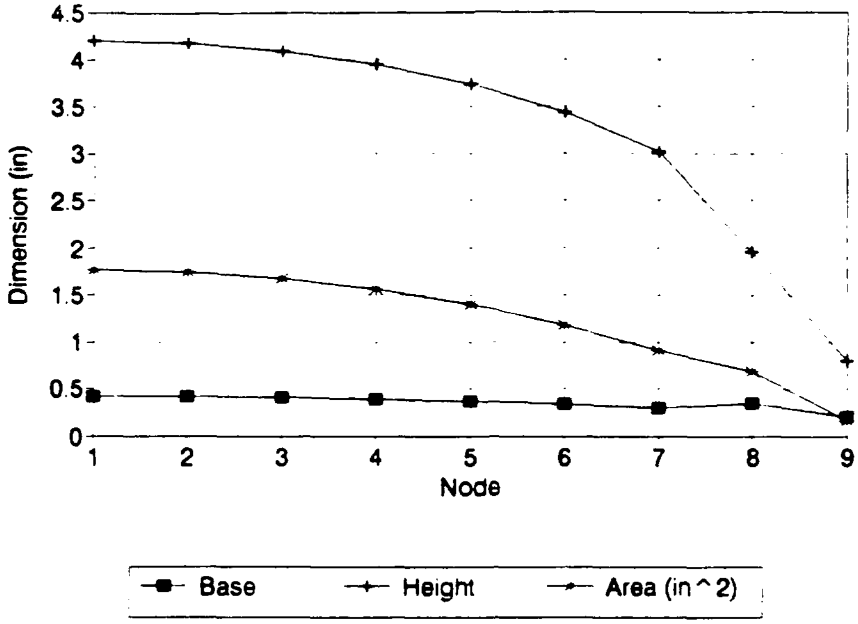
G. CASE #4: CANTILEVER ARCH WITH LATERAL LOADING AND MOMENT

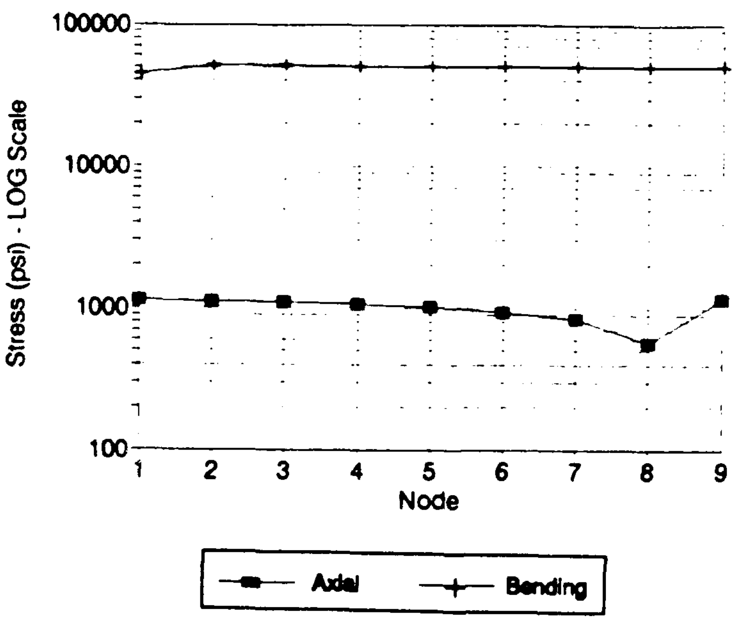
For this case, a lateral load and concentrated moment were applied at nodal point 9. The shape of the dimension plot curves are very similar to those of the cantilever beam, parabolic in form. In comparison with the same structure subject only to the lateral load, Case #2, the total structure volume is reduced by 18.67%. The concentrated end moment negates the effect of lateral load on the extreme fibers by producing compressive stresses on the outer fibers and tensile stresses on the inner fibers of the arch. Thus, the cross sectional dimensions necessary to withstand the total normal stress is reduced thereby reducing the total structure volume.



Loads		
Lateral	=	2,000 lbs
Axial	=	0 lbs
Moment	=	1,000 in-lbs
End conditions		
Node 1		0 MOD
Node 9		3 MOD
Dimensions		
Radius	=	32 in
Theta	=	90 degrees
Total volume		
Volume	=	63.25 in^3

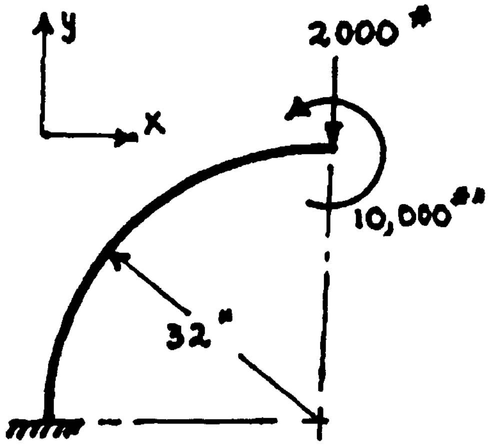
Case #4


4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
Dimension (in)
1 2 3 4 5 6 7 8 9
Node
Base
Height
Area (in^2)


100000
10000
1000
100
Stress (psi) - LOG Scale
1 2 3 4 5 6 7 8 9
Node
Axial
Bending

H. CASE #4A: CANTILEVER ARCH WITH LATERAL LOADING AND MOMENT

To further emphasize the effect of the concentrated end moment, the structure of Case #4 was subject to the same lateral load while the moment at the end point was increased by a factor of 10. By increasing the applied moment, the effect of the lateral load on the extreme fibers is negated further which reduces the cross sectional area necessary to withstand the total stresses. Expectedly, the volume reduced from Case #4 by 15.88% for a total reduction from Case #2 of 31.60%. It is interesting to note that the shape of the dimension curves still remain parabolic in form.



Loads

Lateral	=	2,000 lbs
Axial	=	0 lbs
Moment	=	10,000 in-lbs

End conditions

Node 1	0 MOD
Node 9	3 MOD

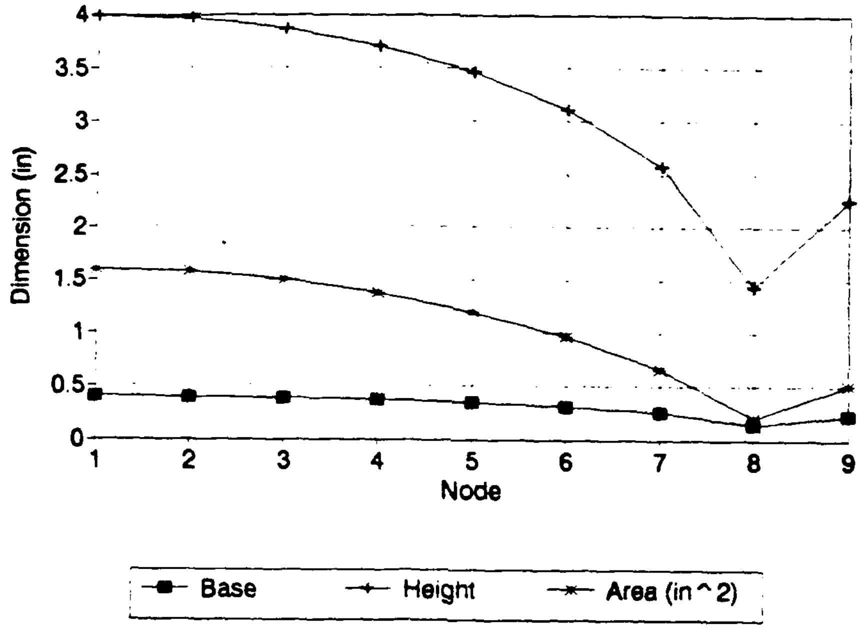
Dimensions

Radius	=	32 in
Theta	=	90 degrees

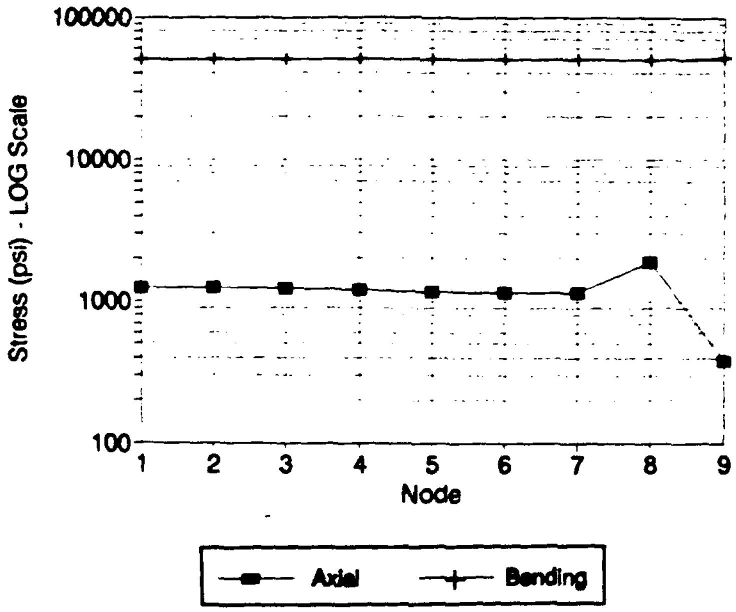
Total volume

Volume	=	53.21 in^3

Case #4a



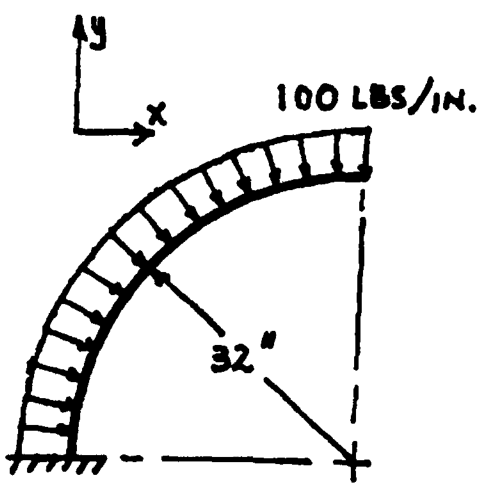
Dimension (in)
4
3.5
3
2.5
2
1.5
1
0.5
0
1 2 3 4 5 6 7 8 9
Node
Base
Height
Area (in^2)



Stress (psi) - LOG Scale
100000
10000
1000
100
1 2 3 4 5 6 7 8 9
Node
Axial
Bending

I. CASE #5: CANTILEVER ARCH WITH DISTRIBUTED LOADING

This case is presented to display some of the versatility of the program. A load acting radially inward is distributed along the length of the arch. The cross section dimensions and area curves appear to be almost linear and the bending stresses dominate the total stresses. Since the bending stress is a function of height squared, the optimizer tried to maximize the height dimension until the geometric constraint was violated. At each nodal point, the height is 10 times the size of the base except at the end point for which both dimensions reach the minimum side constraint. Had the arch structure not been optimized, the volume necessary to support the distributed load would increase by 225%.



Loads

Lateral	=	0 lbs
Axial	=	0 lbs
Moment	=	0 in-lbs
Distrib.	=	1,000 lbs/in

End conditions

Node 1	0 MOD
Node 9	3 MOD

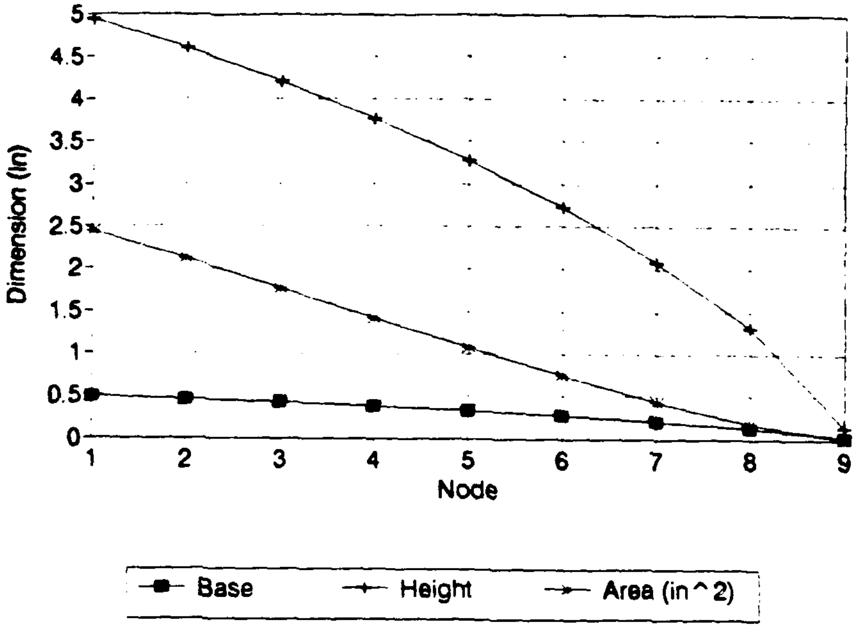
Dimensions

Radius	=	32 in
Theta	=	90 degrees

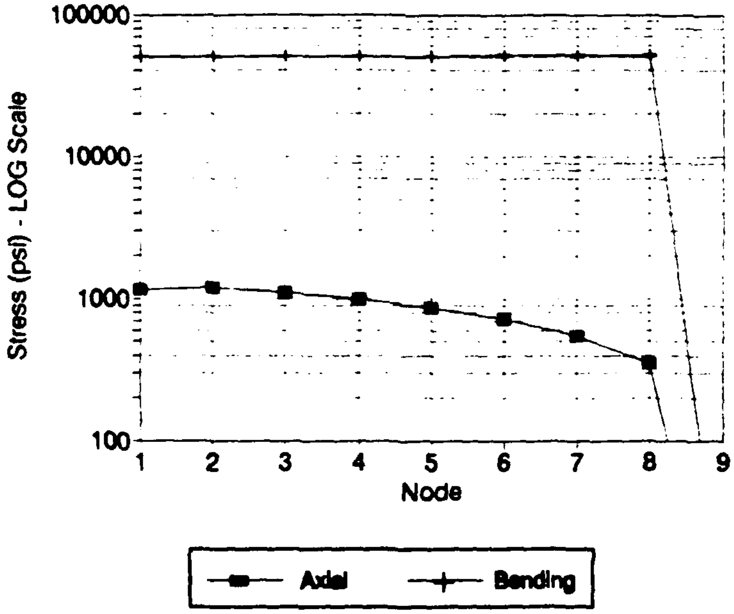
Total volume

Volume	=	55.70 in^3

Case #5



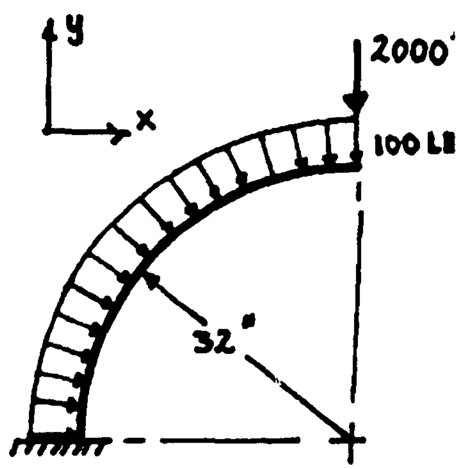
Dimension (in)
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
1 2 3 4 5 6 7 8 9
Node
Base
Height
Area (in^2)



Stress (psi) - LOG Scale
100000
10000
1000
100
1 2 3 4 5 6 7 8 9
Node
Axial
Bending

J. CASE #6: CANTILEVER ARCH WITH LATERAL AND DISTRIBUTED LOADING

To build on Case #5, a lateral load was applied at the end point in addition to the distributed load. In comparison, the volume required to withstand the lateral load only is 77.78 in^3 (Case #2). The volume required to withstand the distributed load only is 55.70 in^3. Yet the volume to withstand both the lateral load and the distributed load presented in this case is 97.47 in^3. By combining loads which produce opposing bending moments, the volume of the resultant optimized arch is not equal to the sum of the volume of arches optimized subject to the individual loads. Therefore, it is possible to achieve a more efficient structure through resourceful combination loadings.



Loads

Lateral	=	2,000 lbs
Axial	=	0 lbs
Moment	=	0 in-lbs
Distrib.	=	100 lbs/in

End conditions

Node 1	0 MOD
Node 9	3 MOD

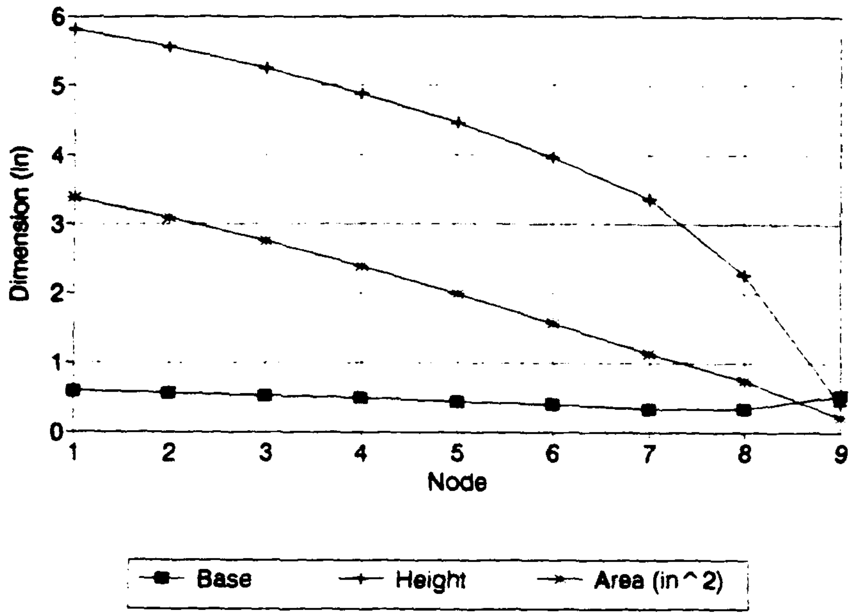
Dimensions

Radius	=	32 in
Theta	=	90 degrees

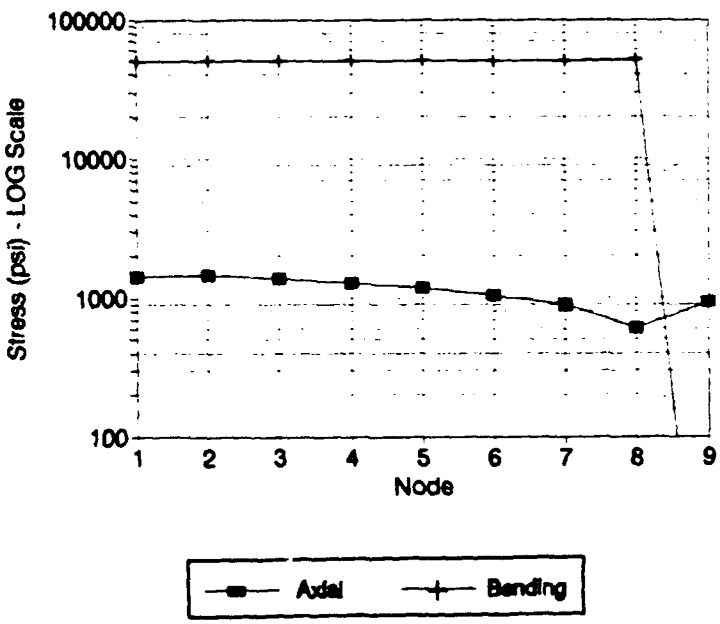
Total volume

Volume	=	97.47 in^3

Case #6



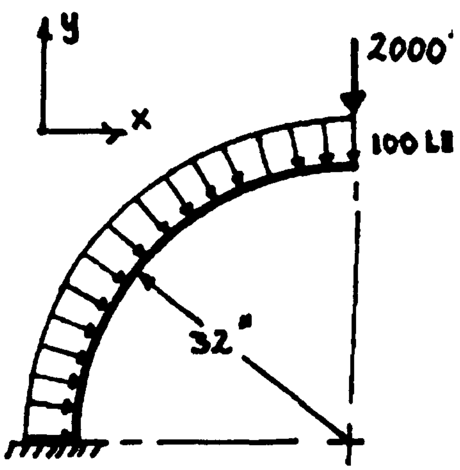
Dimension (in)
6
5
4
3
2
1
0
1
2
3
4
5
6
7
8
9
Node
Base
Height
Area (in^2)



Stress (psi) - LOG Scale
100000
10000
1000
100
1
2
3
4
5
6
7
8
9
Node
Axial
Bending

K. CASE #6A: CANTILEVER ARCH WITH LATERAL AND DISTRIBUTED LOADING

For comparison, the same structure (Case #6) was optimized with the DOT auto scaling function switched on. Changing this parameter seemed to have little effect on the overall volume indicated by an increase by only 4.40%. However, the computation effort judged by total computer time nearly doubled and both the dimension and stress curves have unexpected behavior near the endpoint. This comparison confirmed that better results were achieved by switching the auto scaling function off for these structures.



Loads

Lateral	=	2,000 lbs
Axial	=	0 lbs
Moment	=	0 in-lbs
Distrib.	=	100 lbs/in

End conditions

Node 1	0 MOD
Node 9	3 MOD

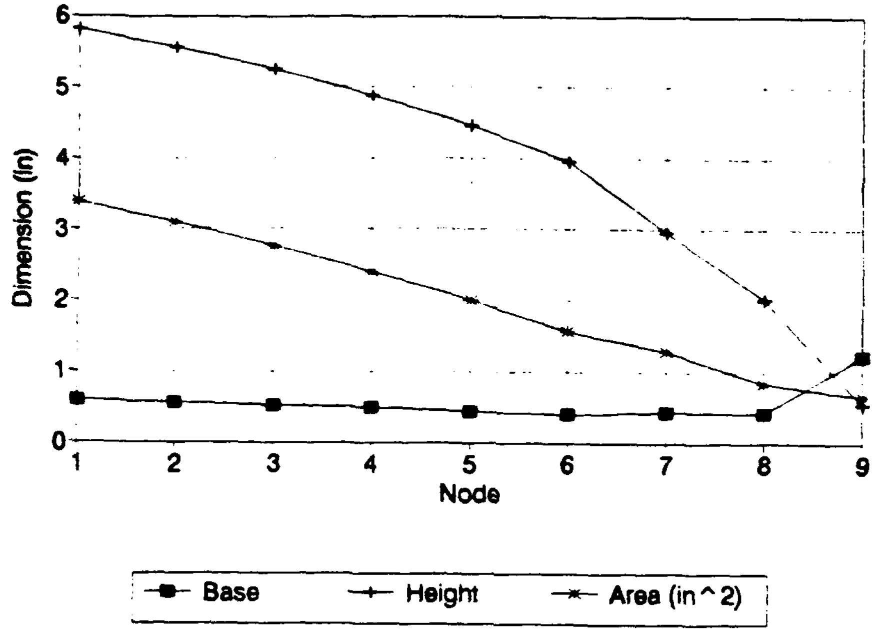
Dimensions

Radius	=	32 in
Theta	=	90 degrees

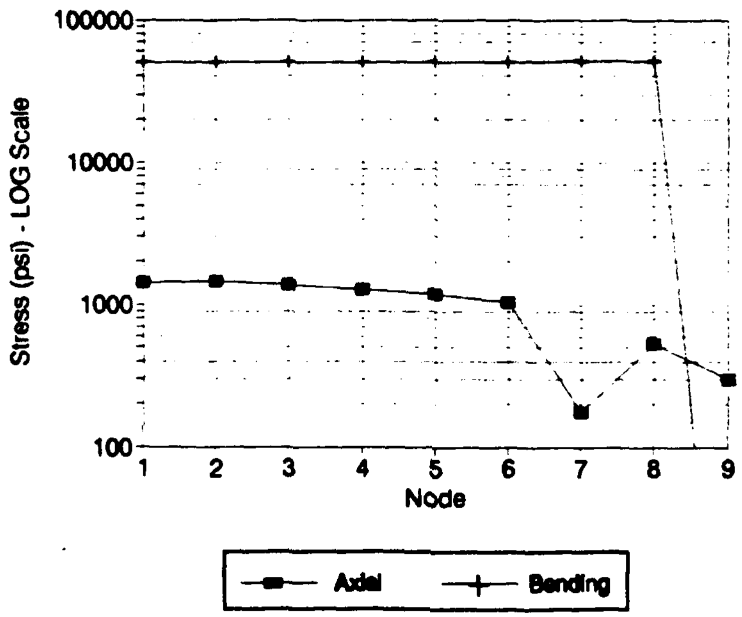
Total volume

Volume	=	101.76 in^3

Case #6a



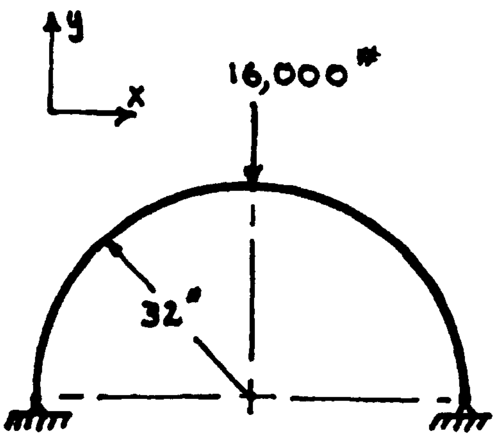
6
5
4
3
2
1
0
Dimension (in)
1 2 3 4 5 6 7 8 9
Node
Base
Height
Area (in^2)



100000
10000
1000
100
Stress (psi) - LOG Scale
1 2 3 4 5 6 7 8 9
Node
Axial
Bending

L. CASE #7: HINGED-HINGED ARCH WITH LATERAL LOADING

For the remaining cases, it was observed that the only reliable and consistent results were obtained by using Method 1 for optimization. It is theorized that restricting displacements at both endpoints may have caused Method 2 to become mathematically unstable and therefore unsuitable to solve such problem. For this particular case, it is interesting to note that at the base, node 1, and 56.25 degrees from the base, node 6, the axial stress completely dominates the total stresses because there is virtually no binding force. At these points, the dimensions of the cross section, dictated strictly by the axial stress, form a square to produce the minimum area.



Loads

Lateral	=	16,000 lbs
Axial	=	0 lbs
Moment	=	0 in-lbs

End conditions

Node 1	1 MOD
Node 9	1 MOD

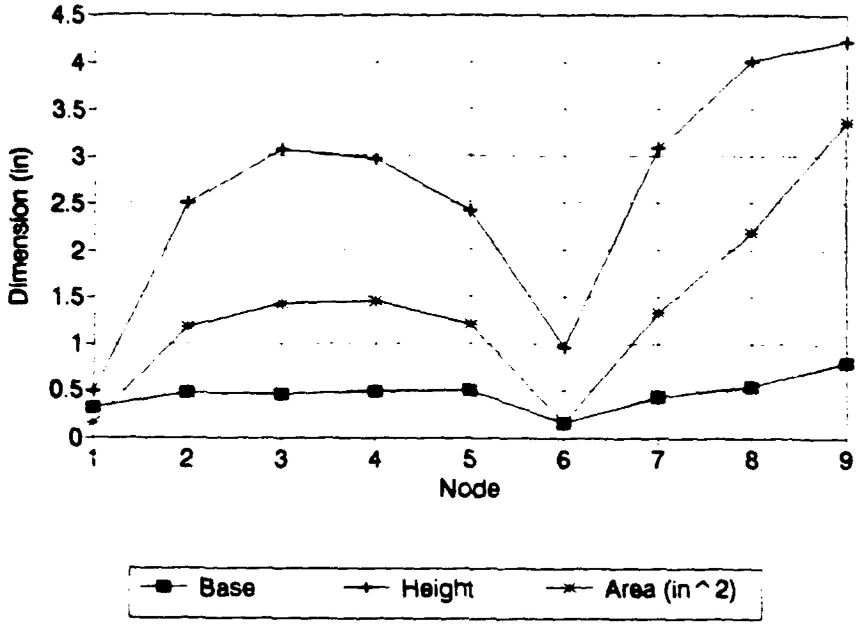
Dimensions

Radius	=	32 in
Theta	=	180 degrees

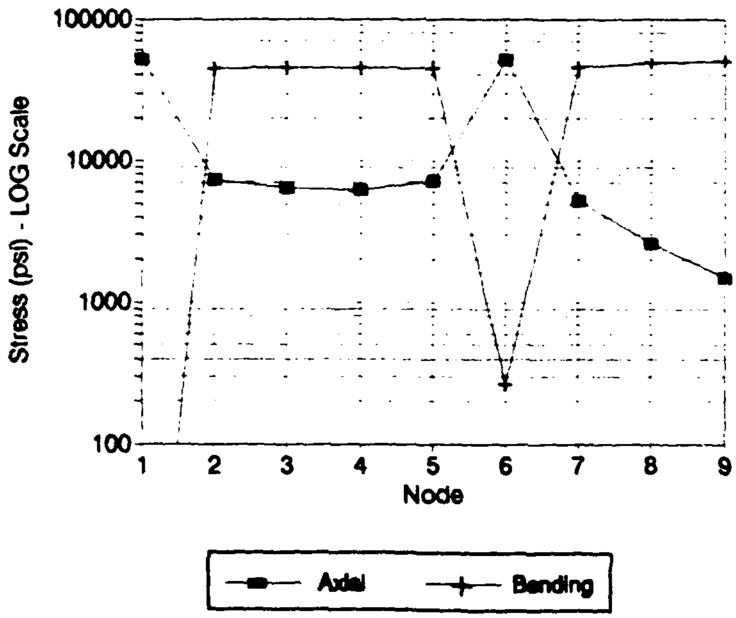
Total volume

Volume	=	129.12 in^3

Case #7



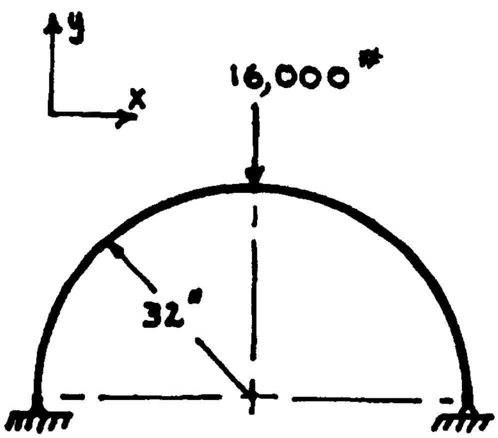
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
Dimension (in)
1
2
3
4
5
6
7
8
9
Node
Base
Height
Area (in ^ 2)



100000
10000
1000
100
Stress (psi) - LOG Scale
1
2
3
4
5
6
7
8
9
Node
Axial
Bending

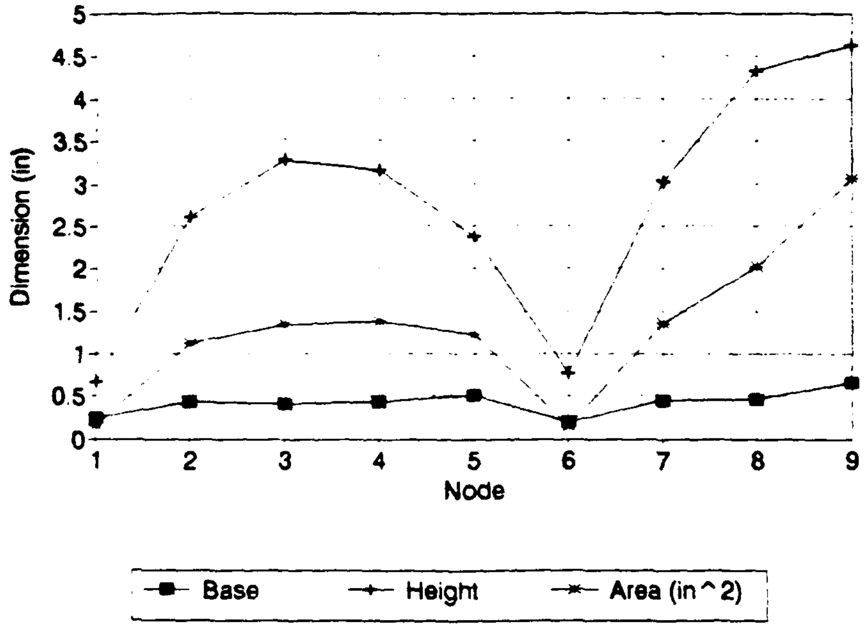
M. CASE #7A: HINGE-HINGED ARCH WITH LATERAL LOADING

Similar to Case #3a, the arch structure of Case #7 was reoptimized using the results achieved as the initial design in order to apply the two-stage optimization strategy. Again, reoptimizing had the desired effect of smoothing the results, however, this effect was not as dramatic for Method 1 as for Method 2. The two-stage optimization strategy only reduced the volume by 4.47% using Method 1 as opposed to the 27.03% reduction using Method 2. Additionally, at node 6, the total stresses exceeded the yield stress by 2.38%. Fortunately, this occurrence did not repeat in any other cases due to reoptimization. Therefore, the two-stage optimization strategy was applied for the remaining cases.

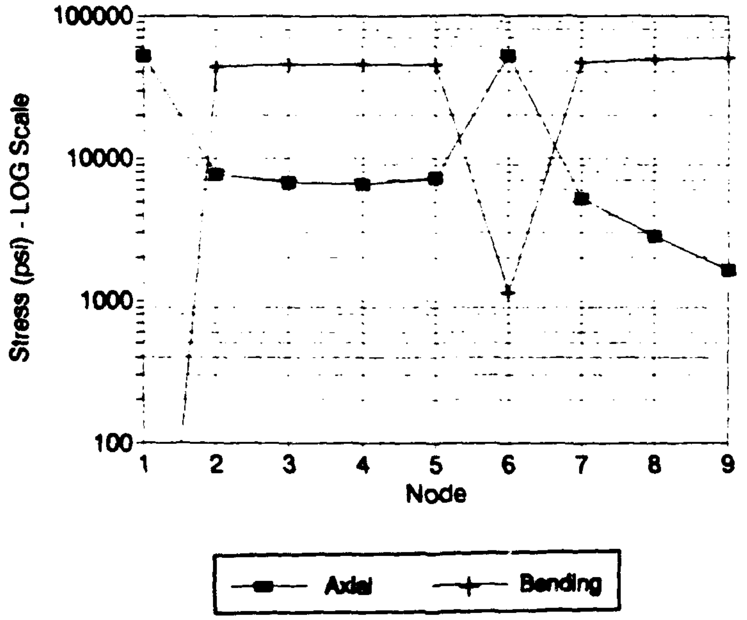


	Loads	
Lateral	=	16,000 lbs
Axial	=	0 lbs
Moment	=	0 in-lbs
	End conditions	
Node 1	1 MOD	
Node 9	1 MOD	
	Dimensions	
Radius	=	32 in
Theta	=	180 degrees
	Total volume	
Volume	=	123.35 in^3

Case #7a



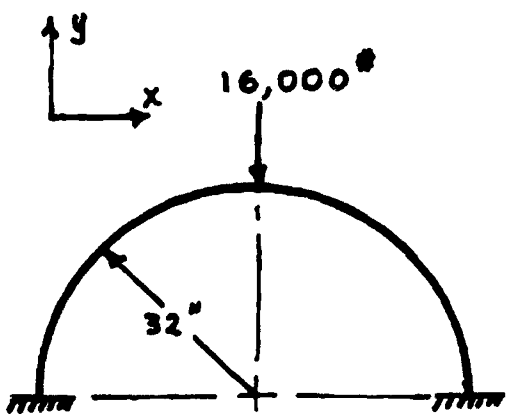
Dimension (in)
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
1
2
3
4
5
6
7
8
9
Node
Base
Height
Area (in^2)



Stress (psi) - LOG Scale
100000
10000
1000
100
1
2
3
4
5
6
7
8
9
Node
Axial
Bending

N. CASE #8: FIXED-FIXED ARCH WITH LATERAL LOADING

For this case, the same loading of Case #7 was applied to a semicircular arch with fixed end points. This produces a statically indeterminate structure with zero means of displacement at both the boundaries. As a result, the peaks of the axial stress curve are dampened and shifted towards the center by approximately 15 degrees. A larger bending moment is produced at the base since it is no longer free to rotate. However, the net results is that the total structure volume of Case #7 is reduced by 14.08% by changing the end conditions from simply-supported to fixed. From this, as expected, a structure more statically indeterminate results in a more efficient structure.



Loads

Lateral	=	16,000 lbs
Axial	=	0 lbs
Moment	=	0 in-lbs

End conditions

Node 1	0 MOD
Node 9	0 MOD

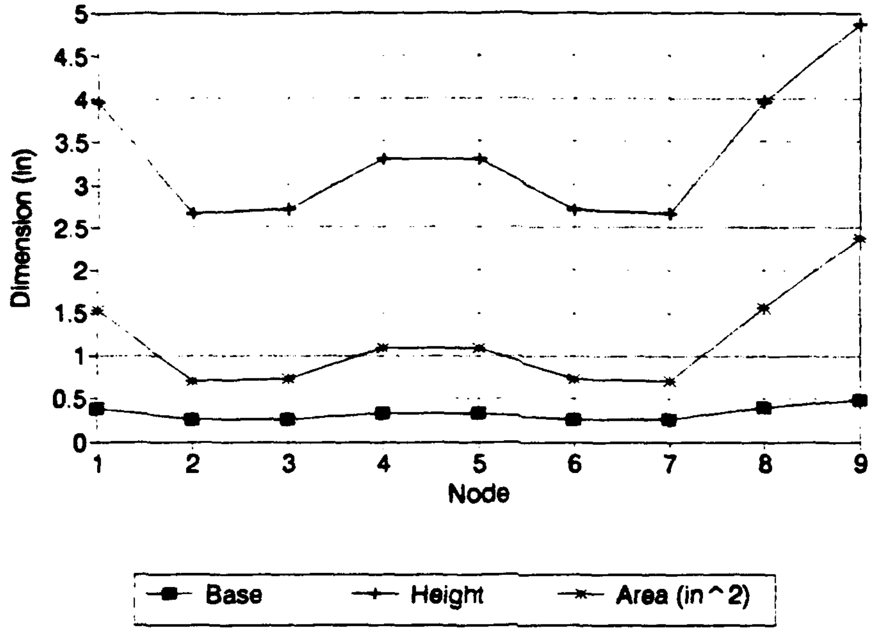
Dimensions

Radius	=	32 in
Theta	=	180 degrees

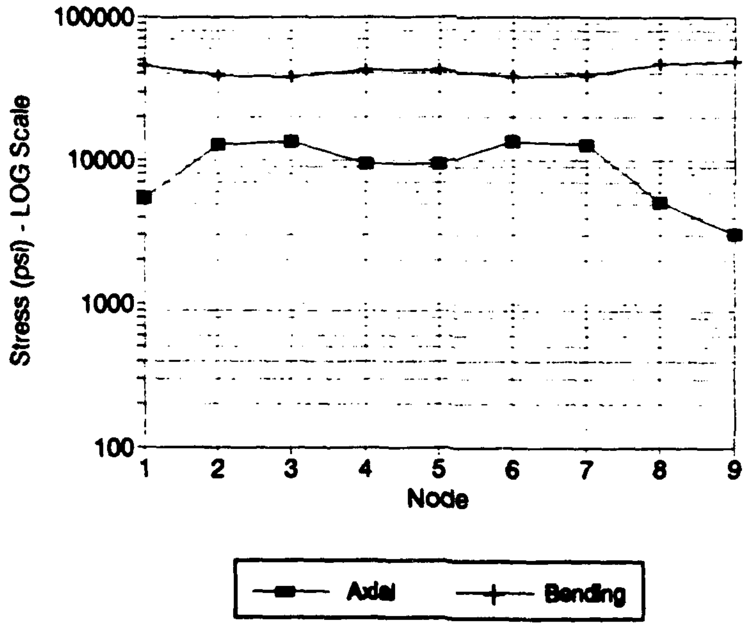
Total volume

Volume	=	105.98 in^3

Case #8



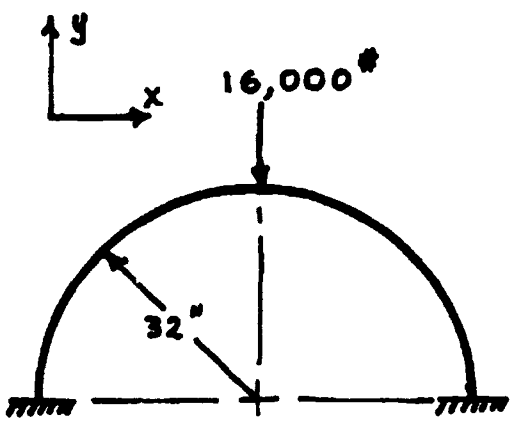
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
Dimension (in)
1 2 3 4 5 6 7 8 9
Node
Base
Height
Area (in^2)



100000
10000
1000
100
Stress (psi) - LOG Scale
1 2 3 4 5 6 7 8 9
Node
Axial
Bending

O. CASE #8A: FIXED-FIXED ARCH WITH LATERAL LOADING

In Chapter I, thesis research performed by Scott McDavid was mentioned as the predecessor for this investigation. His results optimized a fixed-fixed arch subject to the same lateral load with respect to the base dimension only. By holding the height dimension constant, the structure must have twice the volume in order to withstand the loading. For most arch structures, the bending stress is usually the more dominate stress. Therefore, the height dimension has more effect on the total volume than the base dimension because the bending stress is a function of base times height squared. When the bending stress dominates, the optimizer will seek to maximize the height.



Loads

Lateral	=	16,000 lbs
Axial	=	0 lbs
Moment	=	0 in-lbs

End conditions

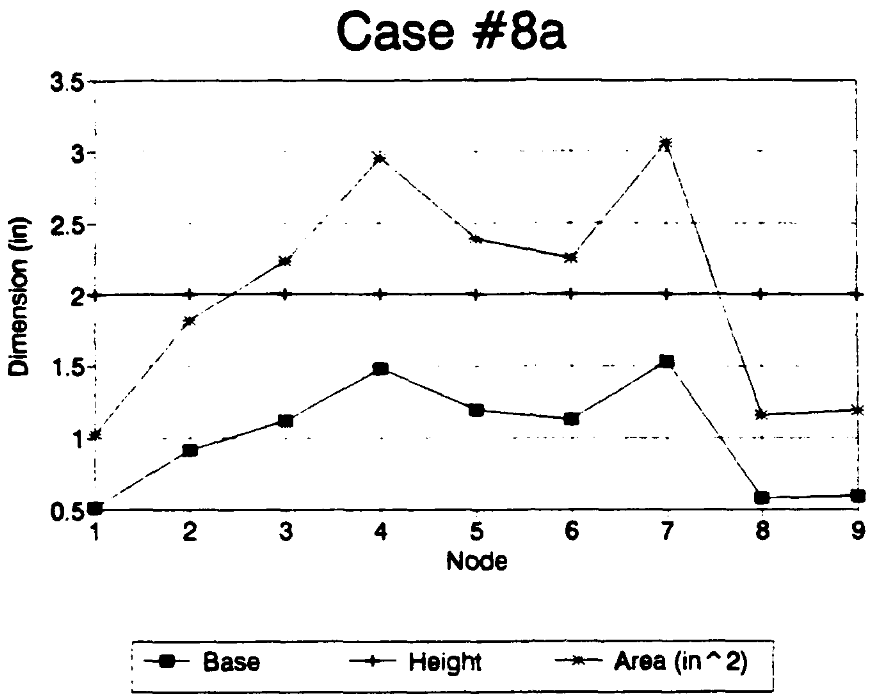
Node 1	0 MOD
Node 9	0 MOD

Dimensions

Radius	=	32 in
Theta	=	180 degrees

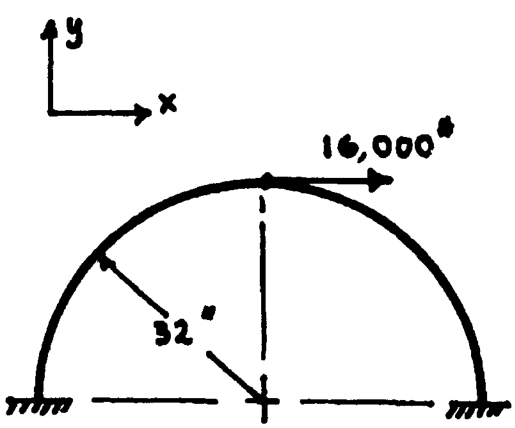
Total volume

Volume	=	215.77 in^3


Case #8a
3.5
3
2.5
2
1.5
1
0.5
Dimension (in)
1
2
3
4
5
6
7
8
9
Node
Base
Height
Area (in^2)

P. CASE #9: FIXED-HINGED ROLLER ARCH WITH AXIAL LOADING

Unable to invoke symmetry on the remaining cases, the elements used to model an asymmetric semicircular arch must span the full 180 degrees. The largest number of elements used to model the structure and produce consistent results remained only eight. It is again suggested that restricting displacements at both endpoints cause the problem to become mathematically unstable. For this particular case, the arch has zero means of displacement at node 1 and two means of displacement at node 9. As expected, the arch is quite large at node 1 to support the resultant moment. At node 3, 45 degrees up from node 1, the axial stress dominates the total stress and the size decreases. For reference, this arch is more than twice the volume of the arches in Case #7 and #8.



Loads

Lateral	=	0 lbs
Axial	=	16,000 lbs
Moment	=	0 in-lbs

End conditions

Node 1	0 MOD
Node 9	2 MOD

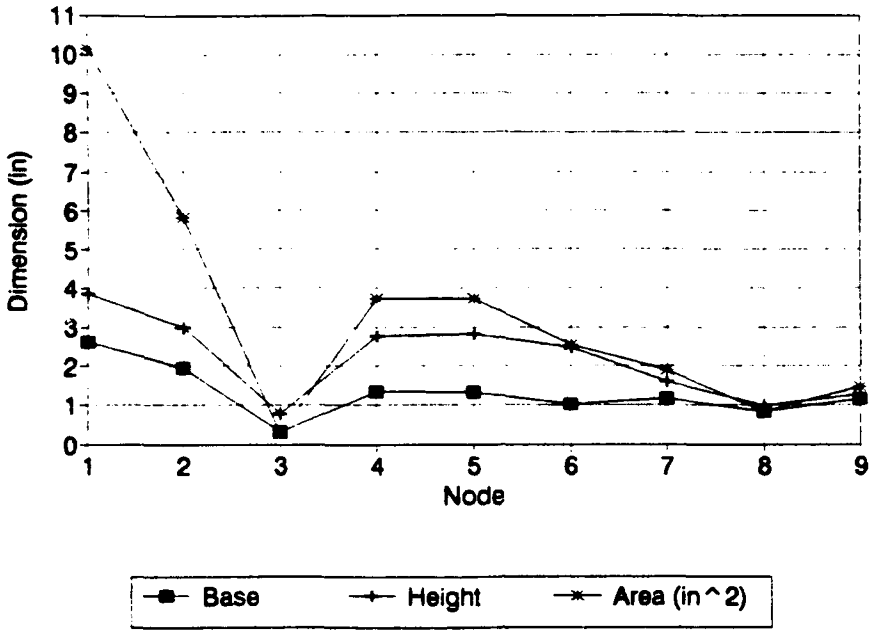
Dimensions

Radius	=	32 in
Theta	=	180 degrees

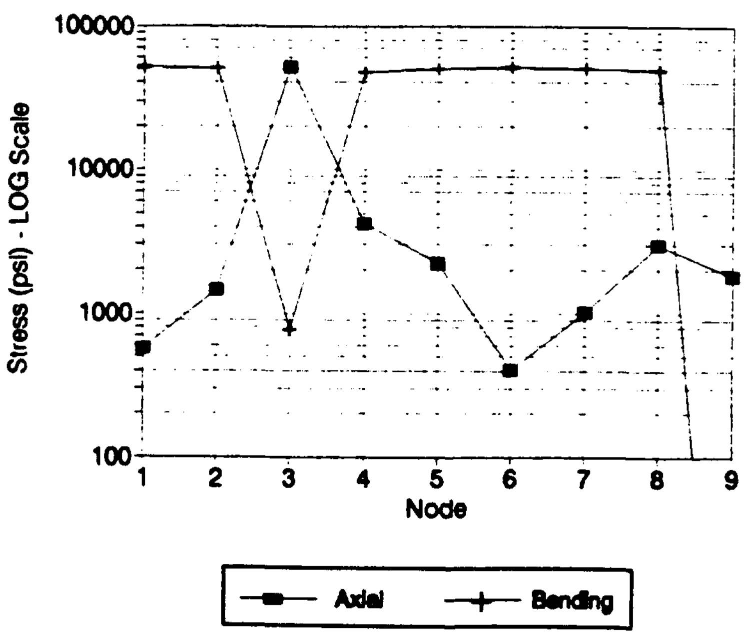
Total volume

Volume	=	287.15 in^3

Case #9



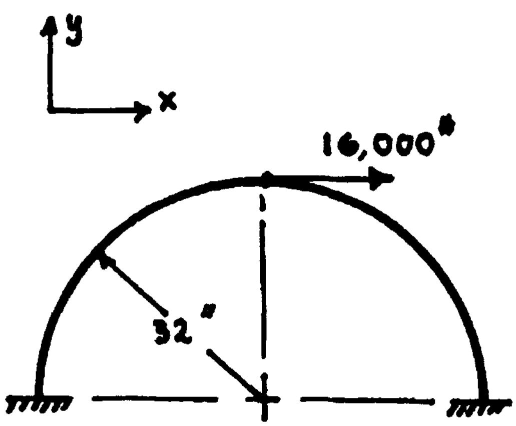
Dimension (in)
Node
Base
Height
Area (in^2)



Stress (psi) - LOG Scale
Node
Axial
Bending

Q. CASE #9A: HINGED-HINGED ROLLER ARCH WITH AXIAL LOADING

To emphasis the conclusion drawn from case #9 about the endpoints, the arch structure and loading studied for Case #9 was modified by adding an additional means of displacement at node 1. Allowing nodal point 1 to rotate freely, the dimension and stress curves alter drastically. The total structure volume increased by 19.92%, yet the structure cannot withstand the stresses. The total stresses exceed the yield stresses by 54.81% resulting in an infeasible design. It appears that the optimizer failed to achieve an optimal solution for this arch structure due to the additional means of displacement.



Loads

Lateral	=	2,000 lbs
Axial	=	16,000 lbs
Moment	=	0 in-lbs

End conditions

Node 1	1 MOD
Node 9	2 MOD

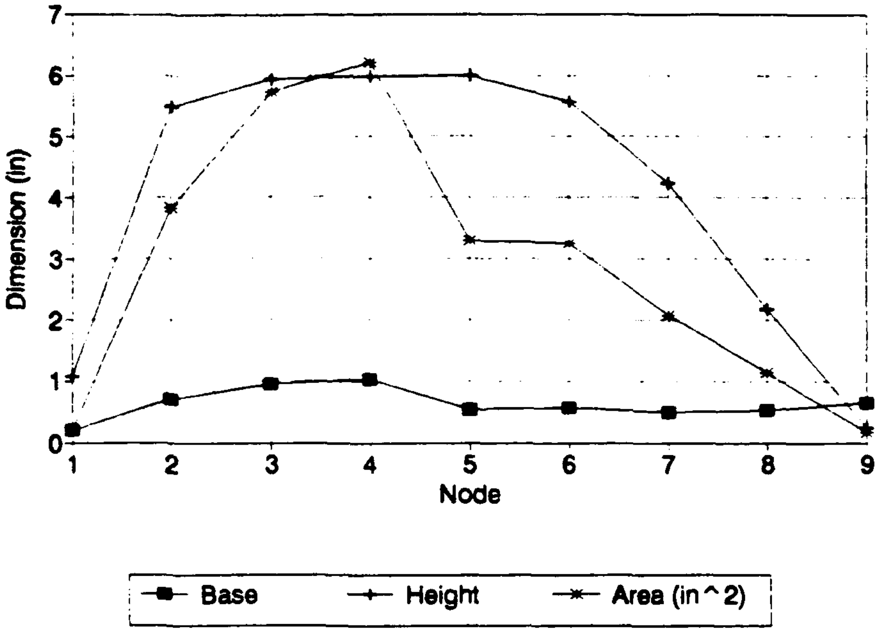
Dimensions

Radius	=	32 in
Theta	=	180 degrees

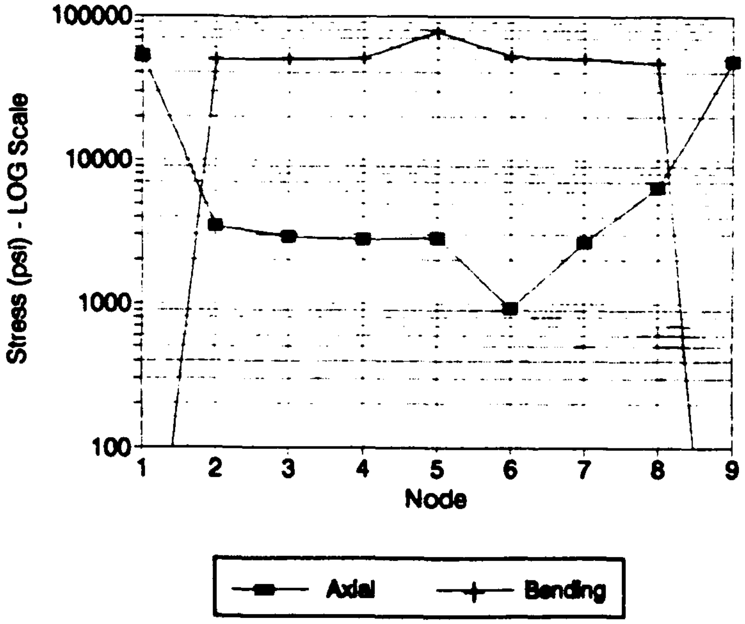
Total volume

Volume	=	344.34 in^3

Case #9a



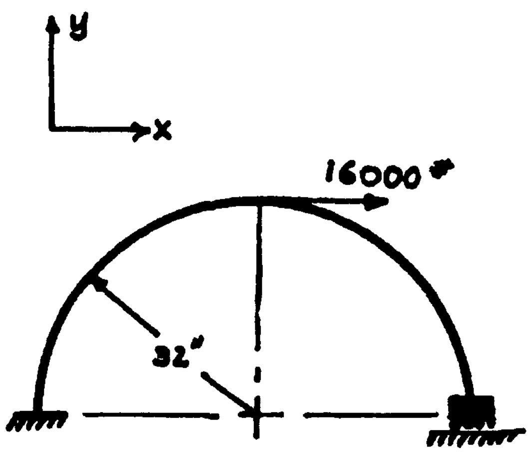
7
6
5
4
3
2
1
0
Dimension (in)
1
2
3
4
5
6
7
8
9
Node
Base
Height
Area (in^2)



100000
10000
1000
100
Stress (psi) - LOG Scale
1
2
3
4
5
6
7
8
9
Node
Axial
Bending

R. CASE #10: FIXED-FIXED ROLLER ARCH WITH AXIAL LOADING

For this case, the same arch structure and loading studied in Case #9 was modified by reducing one means of displacement at node point 9. This produces a more redundant structure with a resultant decrease in total volume of 10.64%. In comparison, Case #8 with one less degree of freedom than Case #7 at both node 1 and node 9, had a reduction in total volume of 14.08%. Again, it is suggested that a structure more statically indeterminate results in a more efficient structure. Additionally, it is noted that when the axial stress dominates the total stresses, the area is reduced significantly and the cross section dimensions reduce to form a square.



Loads

Lateral	=	0 lbs
Axial	=	16,000 lbs
Moment	=	0 in-lbs

End conditions

Node 1	0 MOD
Node 9	1 MOD

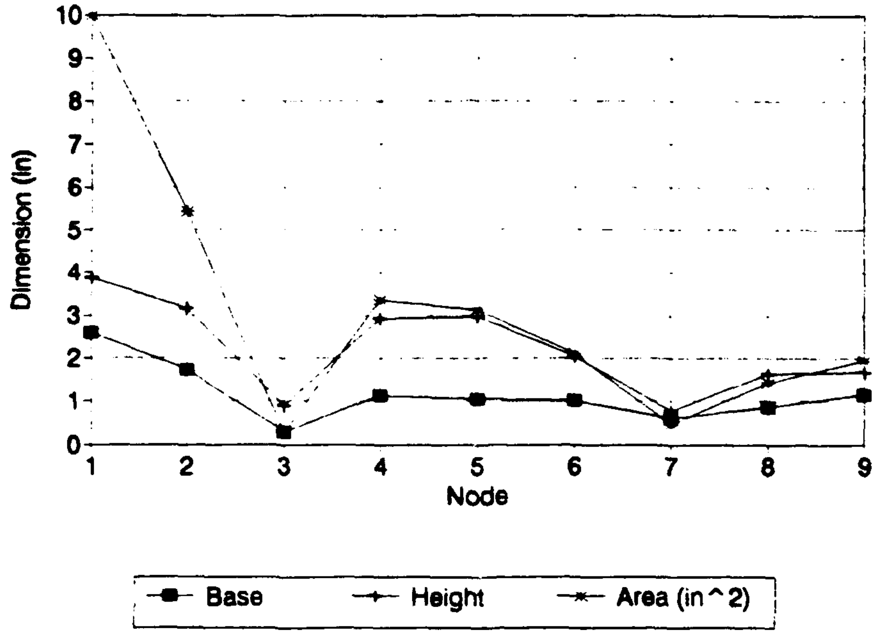
Dimensions

Radius	=	32 in
Theta	=	180 degrees

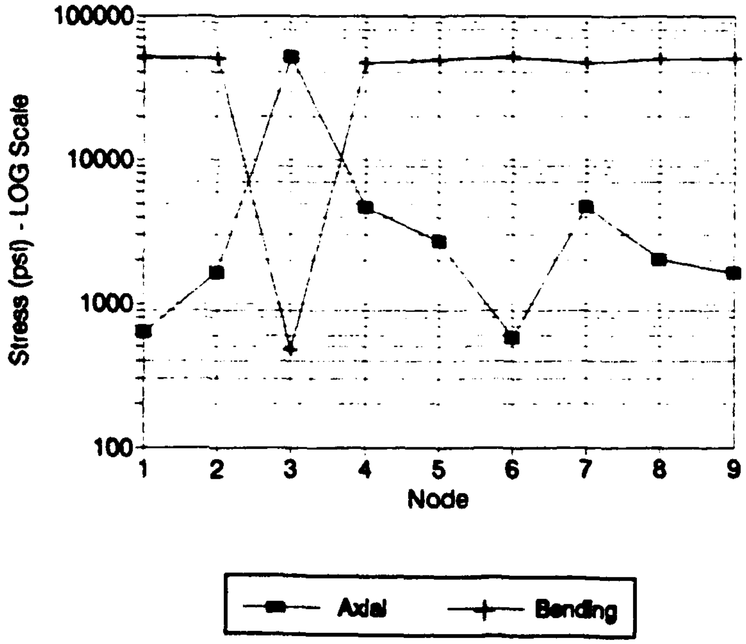
Total volume

Volume	=	256.61 in^3

Case #10



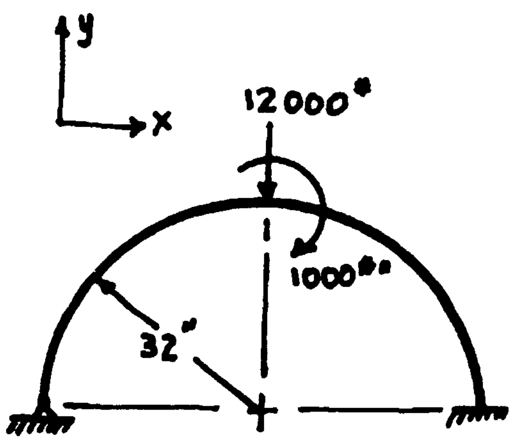
10
9
8
7
6
5
4
3
2
1
0
Dimension (in)
1
2
3
4
5
6
7
8
9
Node
Base
Height
Area (in^2)



100000
10000
1000
100
Stress (psi) - LOG Scale
1
2
3
4
5
6
7
8
9
Node
Axial
Bending

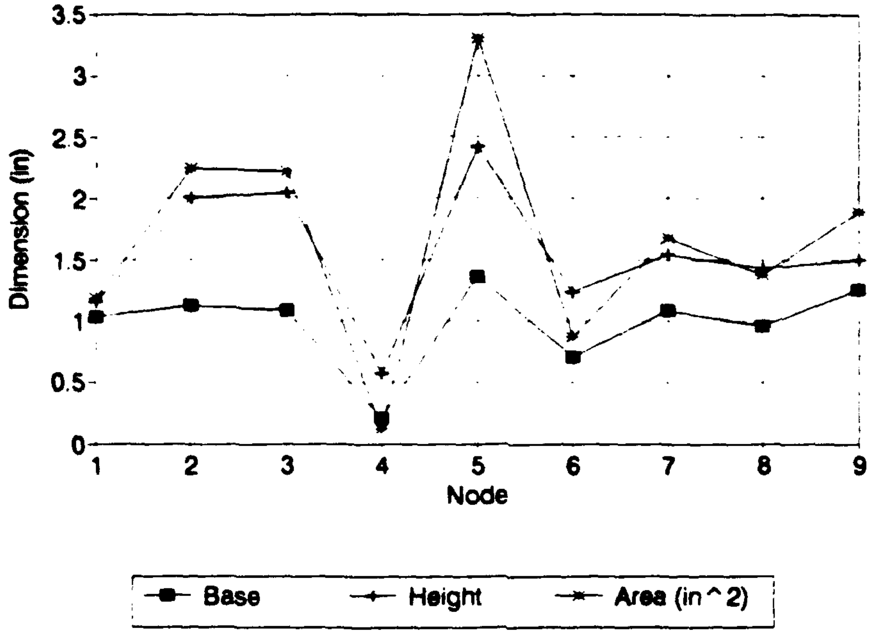
S. CASE #11: HINGED-FIXED ARCH WITH LATERAL LOADING AND MOMENT

To investigate the possibility that dominant axial stresses result in volume reduction, a semicircular arch with one degree of freedom at node 1 and zero degrees of freedom at node 9 was subjected to a lateral load and applied bending moment. From this, it appears that the cross sectional area is inversely proportional to the axial stresses. Additionally, it appears that the dimension and stress curves of the left half of the structure behaves exactly as those of Case #7 which has the identical end conditions. Similarly, the curves of the right half of the arch behaves exactly as those of Case #8. This suggests that the boundary conditions do not effect the structure past the midpoint.

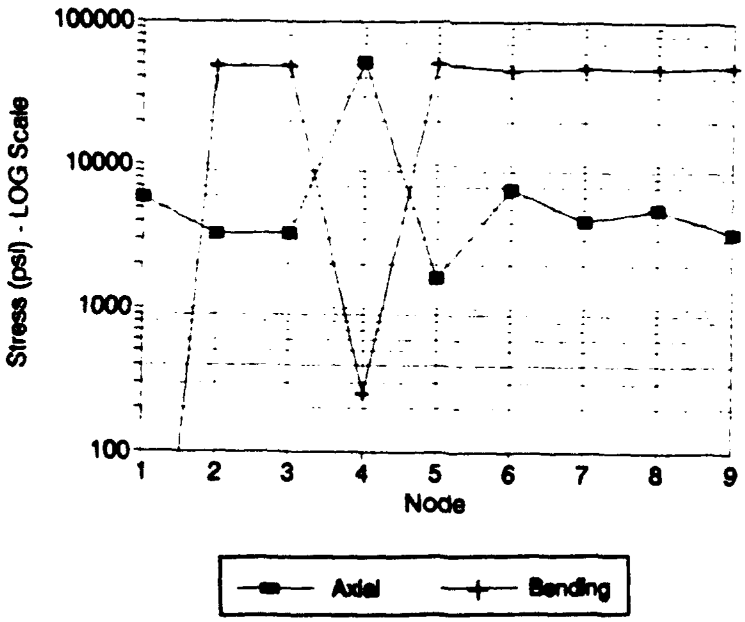


	Loads	
Lateral	=	12,000 lbs
Axial	=	0 lbs
Moment	=	1,000 in-lbs
	End conditions	
Node 1	1 MOD	
Node 9	0 MOD	
	Dimensions	
Radius	=	32 in
Theta	=	180 degrees
	Total volume	
Volume	=	153.08 in^3

Case #11



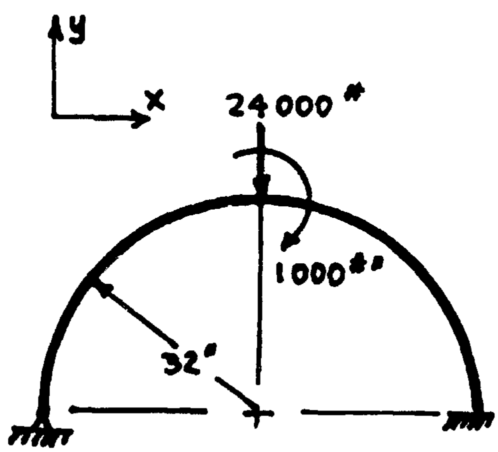
3.5
3
2.5
2
1.5
1
0.5
0
Dimension (in)
1
2
3
4
5
6
7
8
9
Node
Base
Height
Area (in^2)



100000
10000
1000
100
Stress (psi) - LOG Scale
1
2
3
4
5
6
7
8
9
Node
Axial
Bending

T. CASE #11A: HINGED-FIXED ARCH WITH LATERAL LOADING AND MOMENT

In order to test the possibility that dominant axial stresses might reduce the cross section area and hence reduce the total structure volume, the structure of Case #11 was subject to the same bending moment while the lateral load was increased by a factor of 2. As a result, the axial stresses increased overall. Again, it appears that the cross sectional area is inversely proportional to the axial stresses. The dimension and stress curves displayed the same shape as noted before but the total volume increased by 57.95%. Therefore, it was concluded that increasing axial stresses may reduce the cross sectional area at specific nodes but the overall structure volume is not reduced.



Loads

Lateral	=	24,000 lbs
Axial	=	0 lbs
Moment	=	1,000 in-lbs

End conditions

Node 1	1 MOD
Node 9	0 MOD

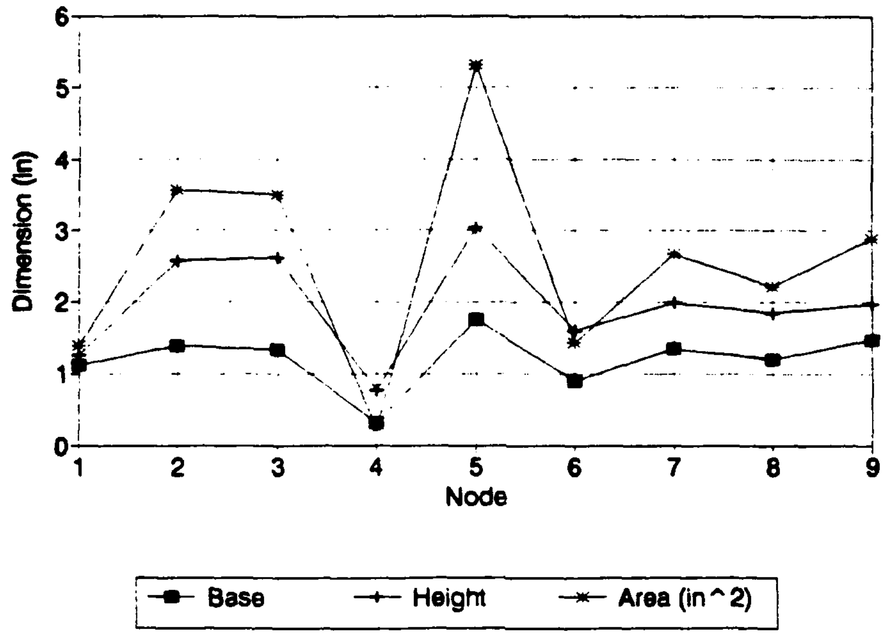
Dimensions

Radius	=	32 in
Theta	=	180 degrees

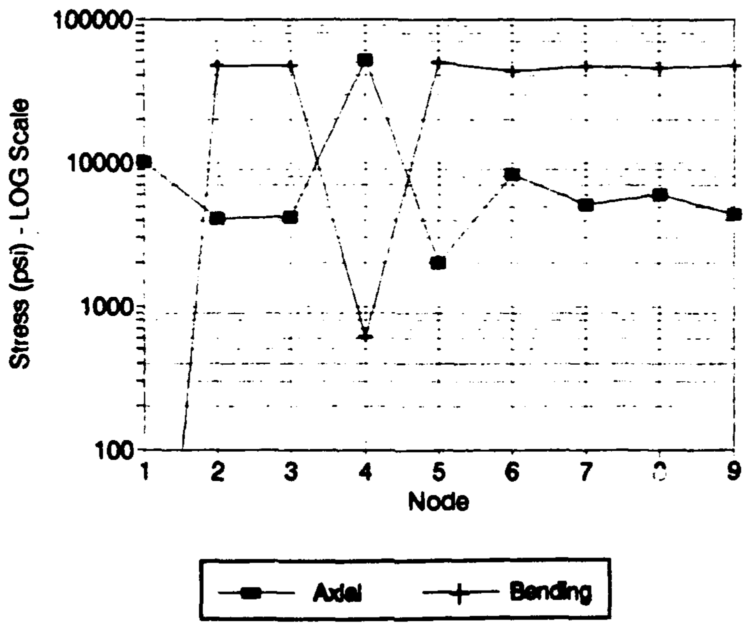
Total volume

Volume	=	241.78 in^3

Case #11a



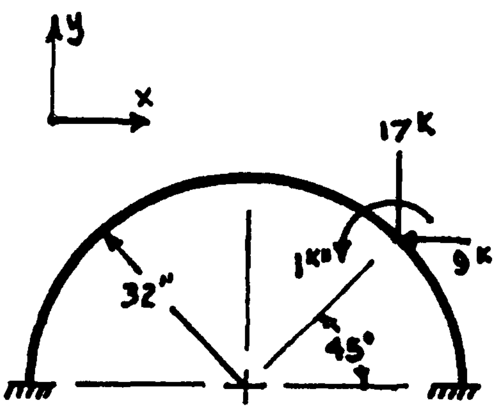
6
5
4
3
2
1
0
Dimension (in)
1 2 3 4 5 6 7 8 9
Node
Base
Height
Area (in^2)



100000
10000
1000
100
Stress (psi) - LOG Scale
1 2 3 4 5 6 7 8 9
Node
Axial
Bending

U. CASE #12: FIXED-FIXED ARCH WITH MULTIPLE LOADING

To demonstrate further versatility of this program, a fixed-fixed arch was subjected to a combination load applied at an angle 45 degrees up from node 9. The load consisted of a concentrated lateral and axial load and an applied bending moment. As anticipated, there is a jump in the dimension curves at node 7 were the load was applied. Interestingly, 22.5 degrees from each endpoint, the axial stress dominates and accordingly, the cross sectional area reduces significantly. Additionally, at node 5, the midpoint of the arch structure, the cross sectional area is significantly smaller as a result of an increase in the axial stress.



Loads

Lateral	=	17,000 lbs
Axial	=	9,000 lbs
Moment	=	1,000 in-lbs

End conditions

Node 1	0 MOD
Node 9	0 MOD

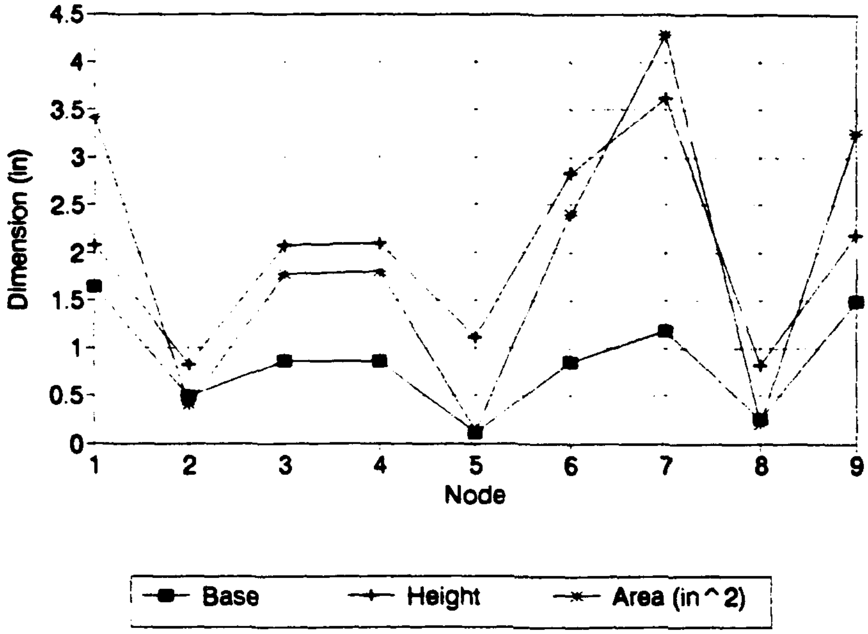
Dimensions

Radius	=	32 in
Theta	=	180 degrees

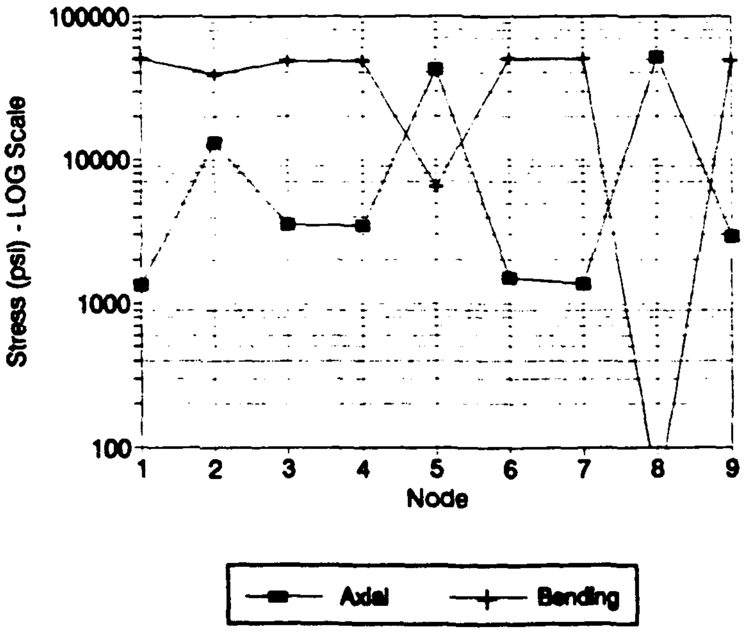
Total volume

Volume	=	156.55 in^3

Case #12



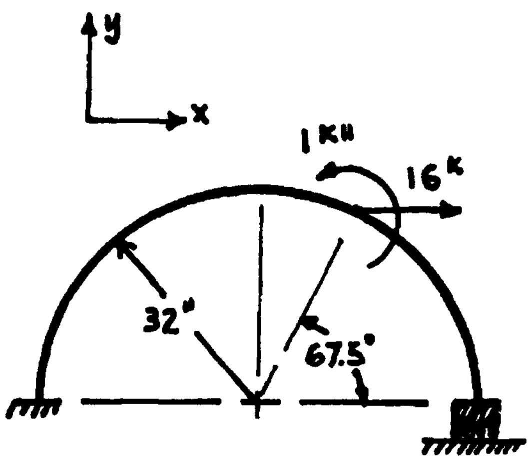
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
Dimension (in)
1
2
3
4
5
6
7
8
9
Node
Base
Height
Area (in^2)



100000
10000
1000
100
Stress (psi) - LOG Scale
1
2
3
4
5
6
7
8
9
Node
Axial
Bending

V. CASE #13: FIXED-FIXED ROLLER ARCH WITH AXIAL LOADING AND MOMENT

For this case, the same structure of Case #10 is subjected to an equivalent axial load with an additional bending moment applied at nodal point 6, 67.5 degrees up from node 9. Shifting the load by 22.5 degrees and adding the applied bending moment appeared to have little effect on the overall design. In fact, the volume is increased from Case #10 by only 3.65% and the dimension curves exhibit very similar characteristics. However, the dip observed previously in the axial stress curve at node 6 of Case #10 is not present in the axial stress curve of this case.



Loads

Lateral	=	0 lbs
Axial	=	16,000 lbs
Moment	=	1,000 in-lbs

End conditions

Node 1	0 MOD
Node 9	1 MOD

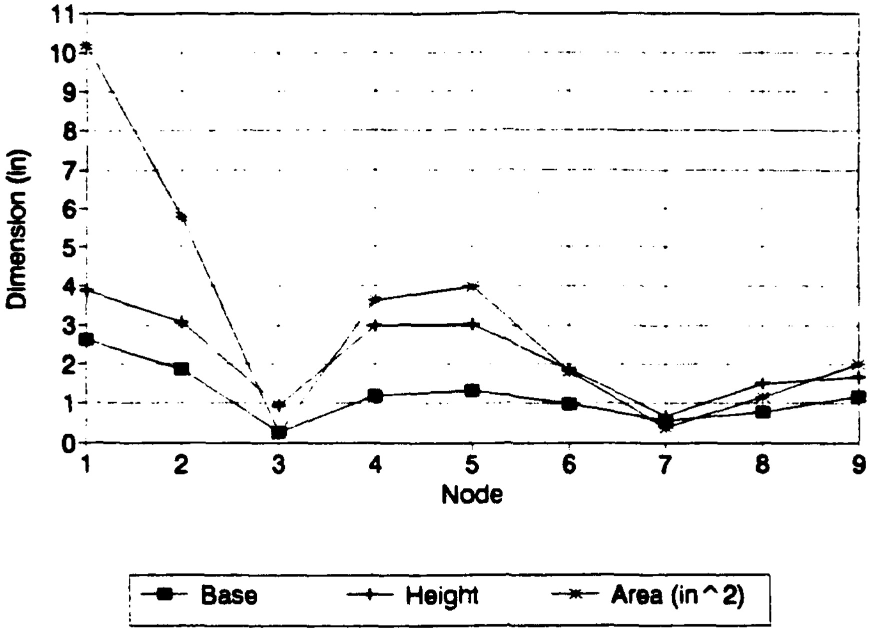
Dimensions

Radius	=	32 in
Theta	=	180 degrees

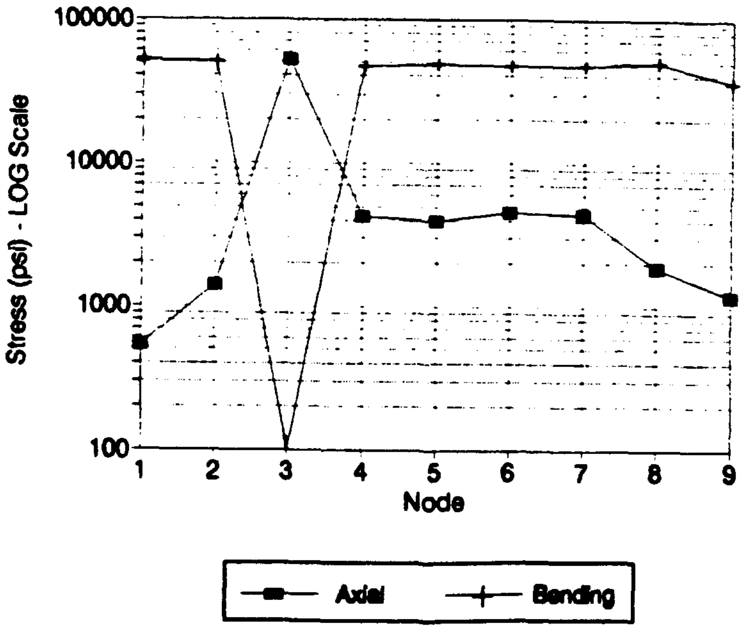
Total volume

Volume	=	265.96 in^3

Case #13



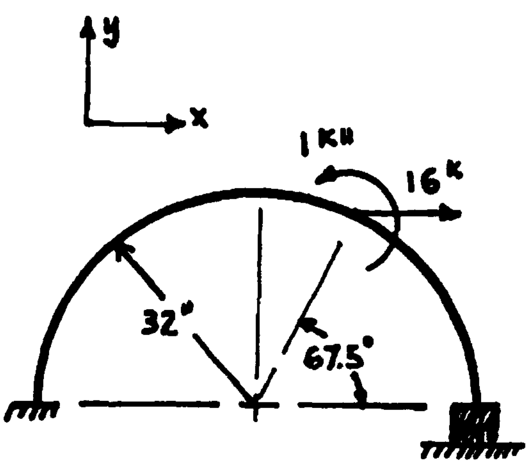
Dimension (in)
0
1
2
3
4
5
6
7
8
9
10
11
1
2
3
4
5
6
7
8
9
Node
Base
Height
Area (in^2)



Stress (psi) - LOG Scale
100
1000
10000
100000
1
2
3
4
5
6
7
8
9
Node
Axial
Bending

W. CASE #13A: FIXED-FIXED ROLLER ARCH WITH AXIAL LOADING AND MOMENT

For comparison, the same structure and loading were reoptimized starting from a different initial design point. Previously, each optimization began from an initial design of 2 inches by 2 inches at each node. For this case, the base dimension at each nodal point was 0.5 inches, and the height dimension at each nodal point was 3.5 inches. Incredibly, the volume of the resultant structure is 33.96% less than the volume of the structure optimized in Case #13. Obviously, optimization can be a function of the starting point. Fortunately in the previous cases, various initial design points were tested and this occurence did not repeat itself.



Loads

Lateral	=	0 lbs
Axial	=	16,000 lbs
Moment	=	1,000 in-lbs

End conditions

Node 1	0 MOD
Node 9	1 MOD

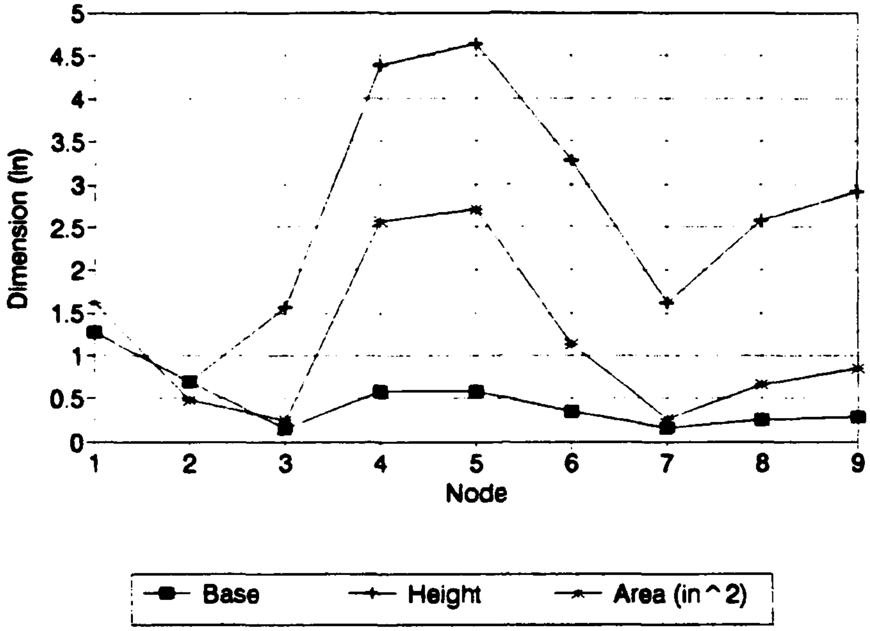
Dimensions

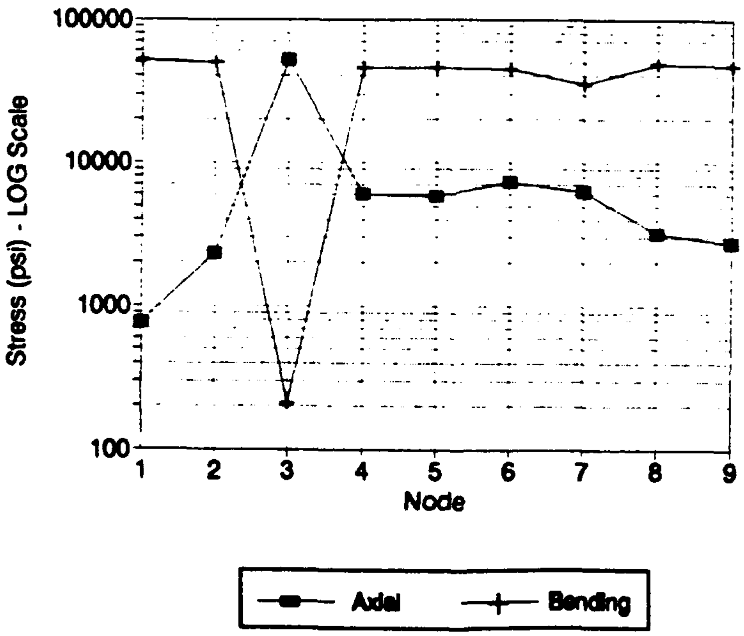
Radius	=	32 in
Theta	=	180 degrees

Total volume

Volume	=	175.65 in^3

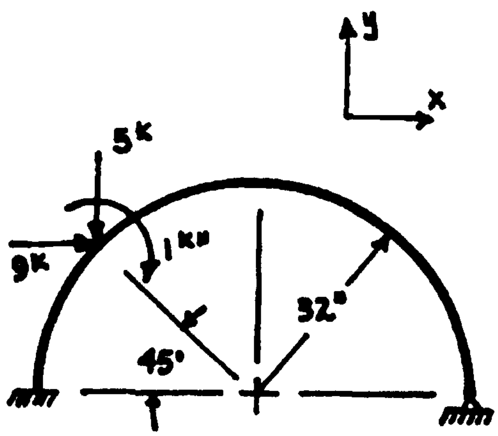
Case #13a


Dimension (in)
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
1 2 3 4 5 6 7 8 9
Node
Base
Height
Area (in^2)


Stress (psi) - LOG Scale
100000
10000
1000
100
1 2 3 4 5 6 7 8 9
Node
Axial
Bending

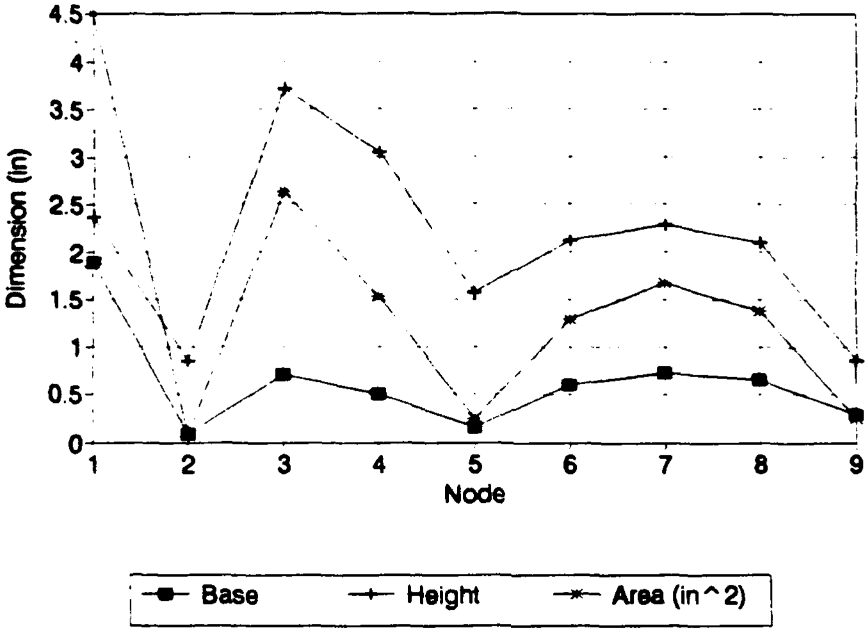
X. CASE #14: FIXED-HINGED ARCH WITH MULTIPLE LOADING

In contrast to Case #13, this asymmetric arch was loaded at an angle on the side of the arch with zero means of displacement at the endpoint. The behavior exhibited by the dimension and stress curves was similar to that of Case #12 which had zero means of displacement at both endpoints. Allowing for the difference in the magnitude and direction of the load, the only significant difference between Case #12 and this case appears at node 8 and 9. It is presumed that the added means of displacement at node 9 caused such a difference. To ensure that a true optimum had been reached, attempts were made to optimize this structure for several different initial starting points. Consistent results were not obtained.

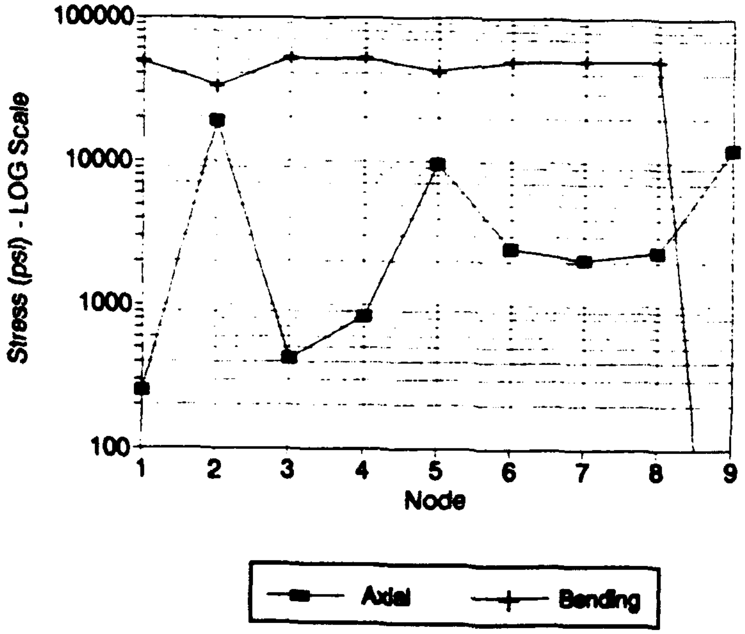


	Loads	
Lateral	=	5,000 lbs
Axial	=	9,000 lbs
Moment	=	1,000 in-lbs
	End conditions	
Node 1	0 MOD	
Node 9	1 MOD	
	Dimensions	
Radius	=	32 in
Theta	=	180 degrees
	Total volume	
Volume	=	121.28 in^3

Case #14



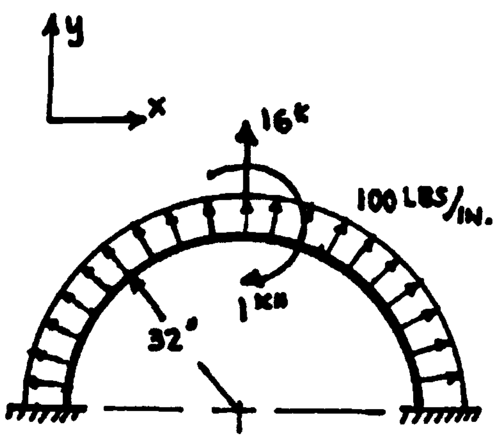
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
Dimension (in)
1 2 3 4 5 6 7 8 9
Node
Base
Height
Area (in^2)



100000
10000
1000
100
Stress (psi) - LOG Scale
1 2 3 4 5 6 7 8 9
Node
Axial
Bending

Y. CASE #15: FIXED-HINGED ROLLER ARCH WITH MULTIPLE LOADING

For the last case studied, a concentrated lateral load and bending moment are applied at the midpoint in combination with a load acting radially outward distributed along the length of the arch. The cross sectional area behaved as anticipated from Case #11, inversely proportional to the axial stress. Again, to ensure that a true optimum had been reached, attempts were made to optimize this structure for several different initial starting points. Consistent results were not obtained.



Loads

Lateral	=	16,000 lbs
Axial	=	0 lbs
Moment	=	1,000 in-lbs
Distrib.	=	100 lbs/in.

End conditions

Node 1	0 MOD
Node 9	2MOD

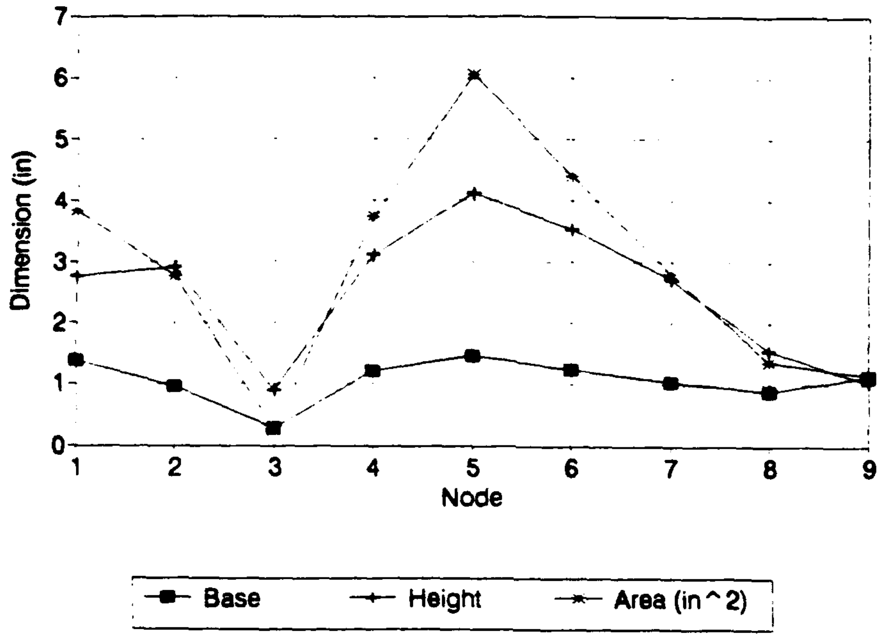
Dimensions

Radius	=	32 in
Theta	=	180 degrees

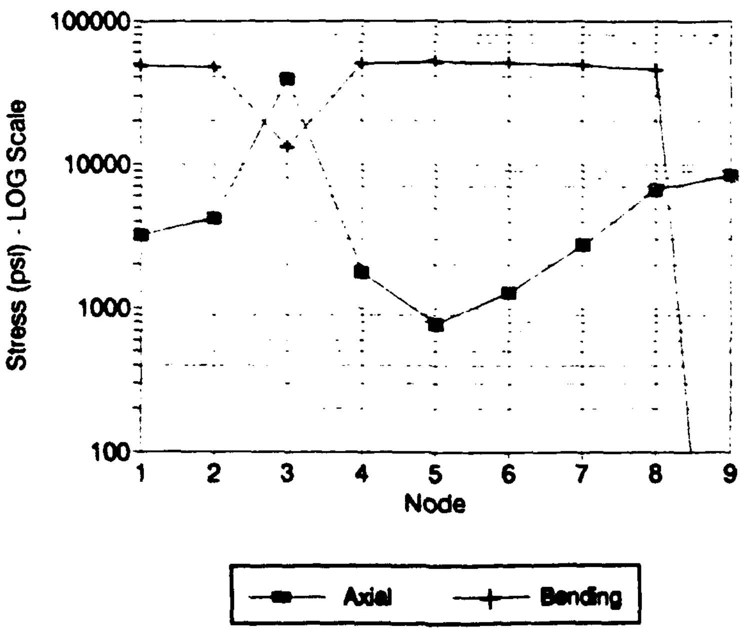
Total volume

Volume	=	285.75 in^3

Case #15



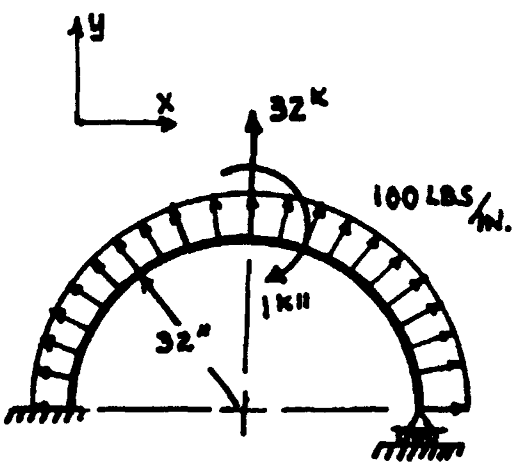
Dimension (in)
7
6
5
4
3
2
1
0
1
2
3
4
5
6
7
8
9
Node
Base
Height
Area (in^2)



Stress (psi) - LOG Scale
100000
10000
1000
100
1
2
3
4
5
6
7
8
9
Node
Axial
Bending

Z. CASE #15A: FIXED-HINGED ROLLER ARCH WITH MULTIPLE LOADING

For comparison, the structure of Case #15 was subject to the same bending moment and distributed load while the lateral load was doubled in value. As demonstrated by Case #11a, an overall increase in the axial stresses results does not effect the shape of the dimension and stress curves. However, the volume from Case #15 is increased by 79.65%. In comparison, doubling the lateral load for Case #11 resulted in an increase in volume of 57.95%. Of interest, it appears that the majority of the volume increase is centered around the midpoint were the increased load was applied.



Loads

Lateral	=	32,000 lbs
Axial	=	0 lbs
Moment	=	1,000 in-lbs
Distrib.	=	100 lbs/in

End conditions

Node 1	0 MOD
Node 9	2 MOD

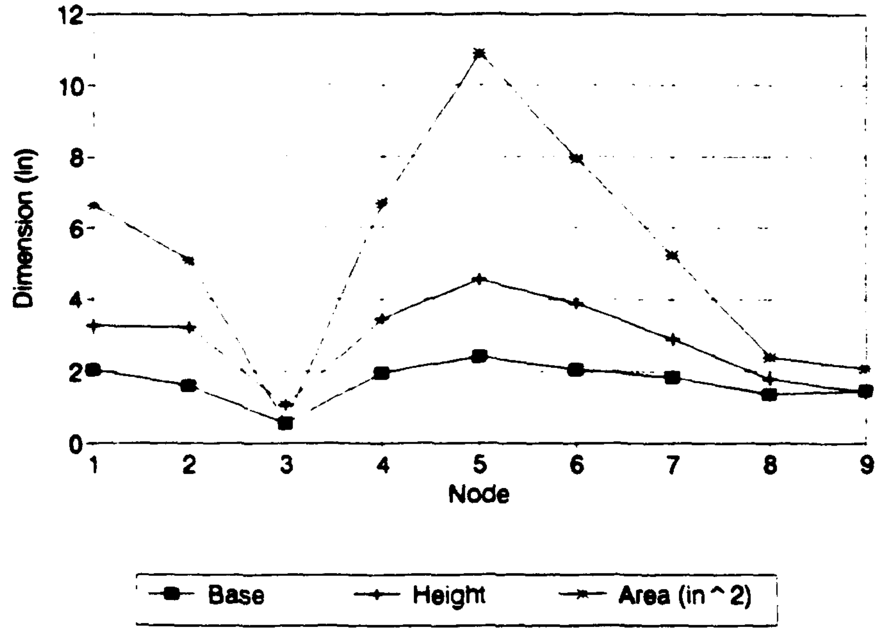
Dimensions

Radius	=	32 in
Theta	=	180 degrees

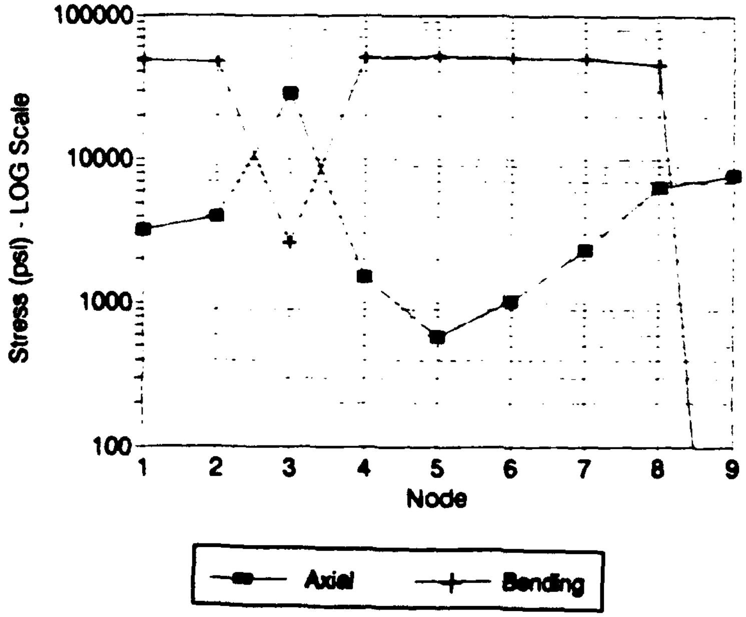
Total volume

Volume	=	516.58 in^3

Case #15a



12
10
8
6
4
2
0
Dimension (in)
1
2
3
4
5
6
7
8
9
Node
Base
Height
Area (in ^ 2)



100000
10000
1000
100
Stress (psi) - LOG Scale
1
2
3
4
5
6
7
8
9
Node
Axial
Bending

VII. CONCLUSIONS

The conclusions of this study are as follows:

- The bar-beam model for stress analysis yielded results which deviated from known analytical solutions with an error of less than 2%. Therefore, the technique of modeling arch structures with bar-beam elements is deemed a viable approximation. (Chapter V)

- From the specific cases studied, the Sequential Linear Programming method, (Method 2), best performed the optimization for cantilever arch structures. The Modified Method of Feasible Directions (Method 1) best performed the optimization for arch structures with restrictive boundary conditions at both endpoints. (Case #2, 2a, 7)

- Reoptimization of an optimal solution has the effect of smoothing the results and reducing the volume of the structure. The effect of this two-stage optimization strategy was more significant for Method 2 than Method 1. (Case #3a, 7a)

- The DOT auto scaling function inhibited the optimizer performance. (Case #6a)

- Applying moments that produce stresses that oppose the stresses produced by a concentrated load reduce the total structure volume required to withstand the combined load. Through prestressing one-way loaded structures, more efficient structures can be achieved. (Case #4, 4a, 6)

- The cross sectional shape is dependant on the type of stress experienced. When bending stresses dominate, the optimal cross section forms a tall rectangle limited only by the geometric constraint. When axial stresses dominate, the optimal cross section dimensions form a square. (Case #5, 7, 10)

- Structures which are more statically indeterminate are more efficient under identical loading than less redundant structures. (Case #8, 10)

- Asymmetric structures are more likely to produce erroneous results due to the limit of the number of elements used to obtain results. (Case #9)

- The boundary conditions act as an excitation which follow the St. Venant principle. The information from the boundary condition diminishes such that for a semicircular arch, the boundary conditions do not effect the cross sectional shape past the arch midpoint. (Case #11, 14)

- Optimization is a function of the initial design starting point. (Case #13a)

From this investigation, the following is suggested as a possibility for future research in the realm of weight optimum arch structures:

- Continue to record results for a comprehensive study of all combinations of parameters, loadings, and end conditions

- Optimize the arch structure using varied cross sections such as a C, L, or I beam, a box beam, or a circular beam.

- Remove the assumption that the arch maintains a constant radius of curvature and optimize the arch shape.

- Apply additional constraints such as global buckling in order to present a more accurate model.

APPENDIX A
DOT PROGRAM PARAMETERS

The information in the following tables is taken from [Ref. 7]

SCALAR PARAMETERS STORED IN RPRM

LOCATION	NAME	DEFAULT VALUE
RPRM(1)	CT	-0.05
RPRM(2)	CTMIN	0.003
RPRM(3)	DABOBJ	MAX[0.001*ABS(F0),0.0001]
RPRM(4)	DELOBJ	0.001
RPRM(5)	DOBJ1	0.1
RPRM(6)	DOBBJ2	0.2*ABS(F0)
RPRM(7)	DX1	0.01
RPRM(8)	DX2	0.2*,AX[X(1)]
RPRM(9)	FDCH	0.001
RPRM(10)	FDCHM	0.0001
RPRM(11)	RMVLMZ	0.4
RPRM(12)	DABSTR	MAX[0.001*ABS(FO),0.00001]
RPRM(13)	DELSTR	0.001
RPRM(14)-RPRM(20) RESERVED FOR INTERNAL USE		

NOTE: FO = The value of the objective function at the start of optimization (for the initial values of X).

DEFINITIONS OF PARAMETERS CONTAINED IN THE RPRM ARRAY

LOC.	PARAM	DEFINITION
1	CT	A constraint is active if its numerical value is more positive than CT. CT is a small negative number
2	CTMIN	A constraint is violated if its numerical value is more positive than CTMIN
3	DABOBJ	Maximum absolute change in the objective between ITRMOP consecutive iterations to indicate convergence in optimization
4	DELOBJ	Maximum relative change in the objective between ITRMOP consecutive iterations to indicate convergence in optimization
5	DOBJ1	Relative change in the objective function attempted on the first optimization iteration. Used to estimate initial move in the one-dimensional search. Updated as the optimization progresses.
6	DOBJ2	Absolute change in the objective function attempted on the first optimization iteration
7	DX1	Maximum relative change in a design variable attempted on the first optimization iteration. Used to estimate the initial move in the one-dimensional search. Updated as the optimization progresses
8	DX2	Maximum absolute change in a design variable attempted on the first optimization iteration. Used to estimate the initial move inthe one-dimensional search. Updated as the optimization progresses.
9	FDCH	Relative finite difference step when calculating gradients
10	FDCHM	Minimum absolute value of the finite difference step when calculating gradients. This prevents too small a step when X(1) is near zero
11	RMVLMZ	Maximum relative change in design variable during the first approximate subproblem in the Sequential Linear Programming Method. This is, each design variable is initially allowed to change by ±40%. This move limit is reduced as the optimization progresses.
12	DABSTR	Maximum absolute change in the objective between itrmst consecutive iterations of the Sequential Linear Programming method to indicate convergence to the optimum
13	DELSTR	Maximum relative change in the objective between ITRMST consecutive iterations of the Sequental Linear Programming method to indicate convergence to the optimum

PARAMETERS IN THE IPRM ARRAY

LOCATION	NAME	DEFAULT VALUE
IPRM(1)	IGRAD	0
IPRM(2)	ISCAL	NDV
IPRM(3)	ITMAX	40
IPRM(4)	ITRMOP	2
IPRM(5)	IWRITE	6
IPRM(6)	NCOLA	NCON+NDV, but at least 2*NDV and not more than 10*NDV
IPRM(7)	IGMAX	0
IPRM(8)	JTMAX	20
IPRM(9)	ITRMST	2
IPRM(10)	JPRINT	0
IPRM(11)	IPRNT1	0
IPRM(12)	IPRNT2	0
IPRM(13	JWRITE	0
IPRM(14)-IPRM(18)		RESERVED FOR FUTURE USE
IPRM(19)	NEWITR	INTERNALLY DEFINED
IPRM(20)	NGT	INTERNALLY DEFINED

DEFINITIONS OF PARAMETERS CONTAINED IN THE IPRM ARRAY

LOC.	PARAM.	DEFINITION
1	IGRAD	Specifies whether the gradients are calculated by DOT (IGRAD=0) or by the user (IGRAD=1)
2	ISCAL	Design variables are rescaled every ISCAL iterations. Set ISCAL=-1 to turn off scaling
3	ITMAX	Maximum number of iterations allowed at the optimize level
4	ITRMOP	The number of consecutive iterations for which the absolute or relative convergence criteria must be met to indicate convergence at the optimizer level
5	IWRITE	File number for printed output
6	NCOLA	Number of columns in constraint gradient matrix A
7	IGMAX	If IGMAX=0, only gradients of active and violated constraints are calucated. If IGMAX>0, up to NCOLA gradients are calculated, including active, violated, and near active constraints
8	JTMAX	Maximum number of iterations allowed for the Sequential Linear Programming method. This is the number of linearized subproblems solved.
9	ITRMST	The number of consecutive iterations for which the absolute or relative convergence criteria must be met to indicate convergence in the Sequential Linear Programming method
10	JPRINT	Sequential Linear Programming subproblem print. If JPRINT>0, IPRINT is turned on during approximate linear subproblem. This is for debugging only
11	IPRNT1	If IPRNT1=1, print scaling factors for the X vector
12	IPRNT2	If IPRNT2=1, print miscellaneous search information. If IPRNT2=2, turn on print during one-dimensional search process. This is for debugging only
13	JWRITE	File number to write iteration history information to. This is useful for using postprocessing program to plot the iteration process. This is only used if JWRITE>0
19	NEWITR	Normally =-1. Set =n at the start of a new iteration, where n is the number of the iteration just completed. If METHOD=0,1, this is after each one-dimensional seaarch. If METHOD=2, this is after each approximate optimization If JWRITE>0, the optimization information will have just been written to that file. If you with to stop after each iternation (or after a particular iteration) and then re-start later, NEWITR is a flag to do this. NEWITR is defined internally by DOT
20	NGT	The number of constraint gradients needed. If the user supplies gradients to DOT, this will be needed. The constraint numbers for which gradients are needed are contained in positiooon 1-NGT of the IWK array. NGT is defined internally by DOT

APPENDIX B
JUSTIFICATION FOR OMITTING SHEAR STRESSES

(The following Appendix is taken from [Ref. 9])

The shear stress distribution through a beam of rectangular cross-section has a parabolic distribution along the height of the member. The maximum shear stress, located at the neutral axis of the beam, is

$$\tau_{max} = 1.5V/A \qquad \textbf{(B.1)}$$

where τ_{max} is the maximum shear stress, V is the shear force, and A is the cross-sectional area of the beam. [Ref. 8]

The normal stress due to bending is given by the equation

$$\sigma_n = Mc/I \qquad \textbf{(B.2)}$$

where σ_n is the maximum normal stress, M is the bending moment, and I is the cross-sectional moment of inertia which for this case is $bh^3/12$ where b and h are the width and height respectively of the cross-section.

Redefining the normal stress in terms of the cross-sectional dimensions yields

$$\sigma_n = M(h/2)/(bh^3/12)$$

or

$$\sigma_n = 6M/hA \qquad \textbf{(B.3)}$$

The ratio of the maximum shear stress to the normal stress due to bending, is denoted by r and given by the expression:

$$r = \tau_{max}/\sigma_n \qquad \textbf{(B.4)}$$

Substituting Equations (B.1) and (B.3) into Equation (B.4) yields

$$r = (1.5V/A) \; / \; (6M/hA)$$

or

$$r = Vh/4M \qquad \textbf{(B.5)}$$

For the cases investigated in this study, the maximum value r can attain is when the loading is that of a uniformly distributed load, p_y. Then, where:

$$V = p_yL \qquad \textbf{(B.6)}$$

$$M = p_yL^2/2 \qquad \textbf{(B.7)}$$

which upon substitution into Equation (B.8) yields

$$r = (p_yL)h/4(p_yL^2/2)$$

which simplifies to

$$r = h/2L \qquad \textbf{(B.8)}$$

The use of the beam equation requires the length of the beam to be at a minimum ten times the height, that is:

$$L \geq 10h \qquad \textbf{(B.9)}$$

To maximize the value of r, let L equal 10h, the minimum allowable length. Substitutiing this value of L into Equation (B.8) yields

$$r \leq h/2(10h)$$

or simply

$$r \leq 1/20 \qquad \textbf{(B.10)}$$

Hence, the maximum shear stress accounts for less than 5% of the bending stress developed in the structure. Five percent is high considering this analysis over-assumed the value of the

shear stress by assigning the maximum shear stress to the entire cross-section of the beam. Moreover, at the outermost fibers where σ_n is a maximum, the shear stress is zero. Therefore, under the circumstances of this study, the addition of shear stresses was deemed to be unwarranted.

APPENDIX C
ARCH_OPTIMIZATION COMPUTER CODE

```
      PROGRAM ARCH_OPTIMIZATION
**********************************************************************
*                                                                    *
*                    ARCH OPTIMIZATION ANALYSIS CODE                 *
*                                                                    *
**********************************************************************
*
*  ALPHA....TRANSFORMATION ANGLE OF ELEMENT (ANGLE TO X-AXIS)
*  ANGLE....TOTAL ANGLE OF ARCH (IN DEGREES)
*  BAVE.....THE AVERAGE BASE DIMENSION ACROSS AN ELEMENT
*  BASE.....DOT ARRAY CONTAINING THE ELEMENTAL BASE DIMENSIONS
*  BASEL....DOT ARRAY CONTAINING THE ELEMENTAL BASE DIMENSIONS LOWER
*           SIDE CONSTRAINT
*  BASEU....DOT ARRAY CONTAINING THE ELEMENTAL BASE DIMENSIONS UPPER
*           SIDE CONSTRAINT
*  BETA ....TRANSFORMATION ANGLE OF ELEMENT (ANGLE TO Y-AXIS)
*  B_1......BOUNDARY TERMS APPLIED AT END "1"
*  B_2......BOUNDARY TERMS APPLIED AT END "2"
*  C1,..,C5.CONSTANTS RELATED TO ELEMENT STIFFNESS COEFFICIENTS
*  CLAN.....CONCENTRATED LOAD APPLICATION NODE (THE NODE FX,FY,FM ARE
*           APPLIED
*  COUNT....COUNTS THE NUMBER OF ITERATIONS COMPLETED
*  DOF......DEGREE OF FREEDOMS (UNKNOWN DISPLACEMENTS & SLOPES)
*  DSN......DESIGN VARIABLE FOR EACH ELEMENT
*  DESIGN...DOT ARRAY CONTAINING THE ELEMENTAL BASE AND HEIGHT DIMENSIONS
*  DESIGNL..DOT ARRAY CONTAINING THE ELEMENTAL BASE AND HEIGHT DIMENSIONS
*           LOWER SIDE CONTRAINT
*  DESIGNU..DOT ARRAY CONTAINING THE ELEMENTAL BASE AND HEIGHT DIMENSIONS
*           UPPER SIDE CONSTRAINT
*  DV1BG....DESIGN VARIABLE #1 (BASE DIMENSION) INITIAL ESTIMATE
*  DV1LO....DESIGN VARIABLE #1 (BASE DIMENSION) LOWER SIDE CONSTRAINT
*  DV1UP....DESIGN VARIABLE #1 (BASE DIMENSION) UPPER SIDE CONSTRAINT
*  DV2BG....DESIGN VARIABLE #2 (HEIGHT DIMENSION) INITIAL ESTIMATE
*  DV2LO....DESIGN VARIABLE #2 (HEIGHT DIMENSION) LOWER SIDE CONSTRAINT
*  DV2UP....DESIGN VARIABLE #2 (HEIGHT DIMENSION) UPPER SIDE CONSTRAINT
*  EK.......6X6 ELEMENT STIFFNESS MATRIX IN LOCAL X,Y COORDINATES
*  EKPR.....6X6 ELEMENT STIFFNESS MATRIX IN ELEMENT LOCAL COORDINATES
*  ELEN.....LENGTH OF ELEMENT
*  F........FORCE VECTOR OF SYSTEM
*  FA.......CONSTANT DISTRIBUTED LOAD OUTWARD FROM END TO END
*  FM.......CONCENTRATED MOMENT AT FREE END
*  FX.......CONCENTRATED LOAD IN X DIRECTION AT FREE END
*  FY.......CONCENTRATED LOAD IN Y DIRECTION AT FREE END
*  G........THE ARRAY OF CONSTRAINT FUNCTIONS
*  GAMMA....6X6 ELEMENT TRANSFORMATION MATRIX
*  GK.......(NDOF)X(NDOF) GLOBAL STIFFNESS MATRIX
*  HAVE.....THE AVERAGE HEIGHT DIM. ACROSS THE ELEMENT
*  HGT......DOT ARRAY CONTAINING THE ELEMENTAL HEIGHT DIMENSIONS
*  HGTL.....DOT ARRAY CONTAINING THE ELEMENTAL HEIGHT DIMENSIONS
*           LOWER SIDE CONSTRAINT
*  HGTU.....DOT ARRAY CONATINING THE ELEMENTAL HEIGHT DIMENSIONS
*           UPPER SIDE CONSTAINT
*  INDSN....INITIAL (UNIFORM) DESIGN DIMENSION
*  INFO.....DOT PARAMETER USED TO SIGNAL THAT THE OPT IS COMPLETE
*  IPRINT...DOT PARAMETER USED SELECT THE DATA OUTPUT FORMAT
*  IPRM.....DOT SELECTABLE INTEGER PARAMETERS
*  ITERATE..THE NUMBER OF TIMES DOT IS TO BE RELOADED WITH THE
*           PRECEEDING DATA
*  IWK......DOT INTERNAL WORK SPACE ARRAY
```

```
*   METHOD...DOT PARAMETER USED TO DEFINE THE OPTIMIZATION METHOD
*   MINMAX...DOT PARAMETER USED TO MINIMIZE/MAXIMIZE THE PROBLEM
*   NCON.....NUMBER OF DESIGN CONSTRAINTS
*   NDOF.....NUMBER OF DEGREES OF FREEDOM
*   NDV......NUMBER OF DESIGN VARIABLES
*   NEL......NUMBER OF ELEMENTS
*   NRIWK....DOT INTERNAL WORK SPACE ARRAY DIMENSION
*   NRWK.....DOT INTERNAL WORK SPACE ARRAY DIMENSION
*   NSNP.....NUMBER OF SYSTEM NODAL POINTS
*   OBJ......THE OBJECTIVE FUNCTION OF THE OPTIMIZATION
*   OPTDCS...OPTIMIZATION DECISION TO OPTIMIZE THE PROBLEM OR NOT
*   P1...P5..PARAMETER DIMENSION CORRESPONDING TO THE NEL, NSNP, NCON,
*            NDOF, AND NDV RESPECTIVELY
*   PHI......SUBTENDED ELELENT ANGLE (ALSO, PHIANG IN DEGREES)
*   PRCSN....THE PRECISION DESIRED TO SOLVE THE FEM SYSTEM OF EQUATIONS
*   RADIUS...ARCH RADIUS
*   RPRM.....DOT SELECTABLE REAL PARAMETERS
*   SIGMA_B..THE ELEMENTAL NORMAL STRESS DUE TO BENDING
*   SIGMA_N..THE ELEMENTAL NORMAL STRESS DUE TO AXIAL FORCES
*   SIGMA_T..THE MAXIMUM TOTAL STRESS IN EACH ELEMENT
*   U........THE "DISPLACEMENT" VECTOR OF THE SYSTEM OF LINEAR EQUATIONS
*   WK.......DOT INTERNAL WORK AREA
*   X........GLOBAL HORIZONTAL COORDINATE
*   Y........GLOBAL VERTICAL COORDINATE
*   YIELD....YIELD STRENGTH OF THE ARCH MATERIAL
*   YOUNG....YOUNG'S MODULUS OF THE ARCH MATERIAL
*************************************************************************
C       ....declare the variables.......................................
        INCLUDE 'ARCH_COM.FOR'
C
C       ....read the input parameters...................................
        OPEN(8, FILE='ARCH_IN.DAT', STATUS='OLD')

        READ(8,*) ANGLE,RADIUS,YOUNG,YIELD,NEL,METHOD,IPRINT,DV1BG,
     &            DV1LO,DV1UP,DV2BG,DV2LO,DV2UP,CLAN,FX,FY,FM,FA,OPTDCS,
     &            ITERATE,PRCSN,BX1,BY1,BM1,BX2,BY2,BM2,LABEL
C
C       ....define constants............................................
        NSNP = NEL + 1
        NDOF = 3*NSNP
        NCON = 3*NSNP
         NDV = 2*NSNP
C
C       ....determine the system nodal coord and element orientation....
        CALL GEOMETRY (NEL,NSNP,ANGLE,RADIUS,X,Y,ALPHA,BETA,ELEN)
C
C       ....define the size of the work arrays for DOT..................
         NRWK = 38800
        NRIWK = 1000
C
C       ....optimize the problem........................................
        CALL OPTIMIZATION_TOOL
C
C       ....compile and format the output...............................
        CALL ARCH_OUTPUT
C
        END
*************************************************************************
*                                                                       *
        SUBROUTINE GEOMETRY (NEL,NSNP.ANGLE,RADIUS,X,Y,ALPHA,BETA,ELEN)
```

```
C       ================================================================
C       | This routine is used by main ARCH_OPTIMIZATION to generate   |
C       | the  x-, y-coordinates of  each system  node, to determine   |
C       | the  orientation  of each  element,  and to  calculate the   |
C       | length of each element.                                      |
C       ================================================================
C       ....declare the variables......................................
        INTEGER NEL,NSNP,P1,P2

        PARAMETER (P1=32,P2=33)

        REAL    ANGLE,RADIUS,ELEN,X(P2),Y(P2),ALPHA(P1),BETA(P1),
     &          PI,PHI,ANG,YNUM,XDEN

        PARAMETER (PI=3.141593)
C
C       ....determine the geometric constants.........................
        PHI = (ANGLE/NEL)*(PI/180.0)
C
        X(1) = 0.0
        Y(1) = 0.0

         ANG = 0.0
C
        DO 100 i=1, NEL
                ANG = ANG + PHI
             X(i+1) = RADIUS * (1.0 - COS(ANG))
             Y(i+1) = RADIUS * SIN(ANG)
               YNUM = (Y(i+1) - Y(i))
               XDEN = (X(i+1) - X(i))
           ALPHA(i) = ATAN2(YNUM,XDEN)
            BETA(i) = (PI/2.0) - ALPHA(i)
  100   CONTINUE
C
C       ....determine the length of each element......................
        ELEN = SQRT(X(2)**2.0 + Y(2)**2.0)
C
        RETURN
        END
*********************************************************************
*                                                                   *
        SUBROUTINE OPTIMIZATION_TOOL
C       ==============================================================
C       This subroutine directs the program flow optimization decision
C       i.e., optimize the problem or not.  It also serves to set up &
C       execute the DOT optimization software.
C       ==============================================================
C       ....declare the variables.....................................
        INCLUDE 'ARCH_COM.FOR'

        INTEGER i
C
C       ....zero out the RPRM and IPRM arrays.........................
        DO 100 i=1,20
           RPRM(i) = 0.0
           IPRM(i) = 0
  100   CONTINUE
C
C       ....initialize COUNT..........................................
        COUNT = 1
```

```
C
C        ....refine the constraint tolerence.........................
         RPRM(2) = 0.0001
         RPRM(3) = 0.001

C
C        ....turn off DOT's auto scaling..............................
         IPRM(2) = -1
C
C        ....increase DOT's default number of iterations...............
         IPRM(3) = 1000
         IPRM(8) = 1000
C
C        ....increase DOT's number of consecutive convergence criteria.
         IPRM(4) = 3
         IPRM(9) = 3
C
C        ....define MINMAX=-1 to minimize the objective function.......
         MINMAX = -1
C
C        ....initialize the design variable limits and best guess......
         DO 200 i=1,NSNP
               BASE(i) = DV1BG
              BASEL(i) = DV1LO
              BASEU(i) = DV1UP
                HGT(i) = DV2BG
               HGTL(i) = DV2LO
               HGTU(i) = DV2UP
  200    CONTINUE
C
C        ....combine base and HGT arrays into design array.........
         DO 250 i=1,NSNP
                j=NSNP+i
             DESIGN(i) = BASE(i)
            DESIGNL(i) = BASEL(i)
            DESIGNU(i) = BASEU(i)
             DESIGN(j) = HGT(i)
            DESIGNL(j) = HGTL(i)
            DESIGNU(j) = HGTU(i)
  250    CONTINUE
C
C        ....make optimization decision..............................

         IF (OPTDCS .NE. 1) THEN
            CALL EVAL
            RETURN
         ENDIF
C
C        ....ready to optimize........................................

         INFO = 0
C
  300    CALL DOT (INFO,METHOD,IPRINT,NDV,NCON,DESIGN,DESIGNL,DESIGNU,
     &                       OBJ,MINMAX,G,RPRM,IPRM,WK,NRWK,IWK,NRIWK)
C
C        ....evaluate the objective function and constraints...........
         IF (INFO .GT. 0) THEN
            CALL EVAL
            GOTO 300
         ENDIF
```

```
C
C       ....refine the solution vector by reoptimizing................
        IF (COUNT .LT. ITERATE) THEN
            INFO = 0
           COUNT = COUNT+1
           GOTO 300
        ENDIF
C
        RETURN
        END
*********************************************************************
*                                                                   *
        SUBROUTINE EVAL
C       =============================================================
C       This subroutine is used to evaluate the Objective function,
C       constraint functions, and side constraints of the optimization
C       problem.
C       =============================================================
C       ....declare the variables.....................................
        INCLUDE 'ARCH_COM.FOR'

        INTEGER i,j
C
C       ....separate the design array into base and HGT arrays
        DO 50 i=1,NSNP
                 j=NSNP+i
             BASE(i) = DESIGN(i)
              HGT(i) = DESIGN(j)
   50   CONTINUE
C
C       ....calculate the objective function..........................
        OBJ = 0.0
C
        DO 100 i=1,NEL
           BAVE(i) = (BASE(i)+BASE(i+1))/2.0
           HAVE(i) = (HGT(i)+HGT(i+1))/2.0
               OBJ = OBJ + BAVE(i)*HAVE(i)*ELEN
  100   CONTINUE
C
C       ....initialize the design constraint vector...................
        DO 200 i=1,NCON
           G(i) = 0.0
  200   CONTINUE
C
C       ....determine the design constraints..........................
        CALL ARCH_STRESS
C
        DO 210 i=1,NSNP
                  j=i+NSNP
                     k=i+(2*NSNP)
           G(i) = (SIGMA_T(i)/YIELD) - 1.0
           G(j) = BASE(i)-(3.0*HGT(i))
           G(k) = HGT(i)-(10.0*BASE(i))
  210   CONTINUE
C
        RETURN
        END
*********************************************************************
*                                                                   *
        SUBROUTINE ARCH_STRESS
```

```
C       ==============================================================
C       This subroutine is used to perform the Finite Element analysis
C       of the stresses developed in an arch or beam for a given load-
C       ing.
C       ==============================================================
C       ....declare the variables.....................................
        INCLUDE 'ARCH_COM.FOR'

        INTEGER IPVT(99)

        REAL    F(P4)

        REAL*8  BK(P4,P4),BF(P4),BU(P4),FAC(9801),WORK(99)

C
C       ....form the element and system matrices......................
        CALL FORM
C
C       ....form the Force vector, F..................................
        CALL FORCE_VECTOR (NEL,NDOF,ELEN,ALPHA,BETA,FA,F)
C
C       ....set the boundary conditiona and loads.....................
        CALL BNDARY (NDOF,GK,CLAN,FX,FY,FM,F,BX1,BY1,BM1,BX2,BY2,BM2)
C
C       ....solve the system of equations.............................
        IF (PRCSN .EQ. 2) THEN
C           ....change GK and F arrays to double precision........
            CALL UPSCALE (NDOF,GK,F,BK,BF)
C           ....solve the system of equations.....................
            CALL DL2ARG (NDOF,BK,P4,BF,1,BU,FAC,IPVT,WORK)
C           ....change BU array to single presicion...............
            CALL DOWNSCALE (NDOF,BU,U)
        ELSE
C           ....solve the system of equations.....................
            CALL L2ARG (NDOF,GK,P4,F,1,U,FAC,IPVT,WORK)
        ENDIF
C
C       ....determine the stress distribution.........................
        CALL STRESS
C
        RETURN
        END
******************************************************************
*                                                                *
        SUBROUTINE FORM
C       ==============================================================
C       This subroutine is used to construct the global stiffness mat-
C       rix for the arch problem.
C       ==============================================================
C       ....declare the variables.....................................
        INCLUDE 'ARCH_COM.FOR'

        INTEGER IEL,I,J,K,II,JJ,KK,III,JJJ

        REAL    C1,C2,C3,C4,C5,CA,CB,EK(P1,6,6),GAMMA(6,6),EKGA(6,6),
     &          GAEKGA(6,6),BH,BH3

C
C       ....define the constants Cx...................................
        C1 = YOUNG/ELEN
```

```
            DO 220 I = 1,6
               DO 215 J = 1,6
                  DO 210 K = 1,6
                     EKGA(I,J) = EKGA(I,J) + EKPR(IEL,I,K)*GAMMA(K,J)
  210             CONTINUE
  215          CONTINUE
  220       CONTINUE
C
C        ....determine the GAEKGA array..............................
            DO 240 I = 1,6
               DO 235 J = 1,6
                  DO 230 K = 1,6
                     GAEKGA(I,J) = GAEKGA(I,J)+GAMMA(K,I)*EKGA(K,J)
  230             CONTINUE
  235          CONTINUE
  240       CONTINUE
C
C        ....copy the GAEKGA array into the EK array..................
            DO 260 I = 1,6
               DO 250 J = 1,6
                  EK(IEL,I,J) = GAEKGA(I,J)
  250          CONTINUE
  260       CONTINUE
  120    CONTINUE
C
C        ....initialize the GK array..................................
         DO 150 I = 1, NDOF
            DO 140 J = 1, NDOF
               GK(I,J) = 0.0
  140       CONTINUE
  150    CONTINUE
C
C        ....construct the GK matrix..................................
         DO 300 IEL = 1, NEL
                 II = 3*(IEL-1)
            DO 290 J = 1, 6
                  JJ = II + J
                  DO 280 K = 1, 6
                        KK = II + K
                     GK(JJ,KK) = GK(JJ,KK)+EK(IEL,J,K)
  280             CONTINUE
  290       CONTINUE
  300    CONTINUE
C
         RETURN
         END
*********************************************************************
*                                                                   *
         SUBROUTINE FORCE_VECTOR (NEL,NDOF,ELEN,ALPHA,BETA,FA,F)
C        ============================================================
C        This subroutine is used to construct the force vector for the
C        FEM problem specified.
C        ============================================================
C        ....declare the variables....................................
         INTEGER NEL,NDOF,i,I1,I2,I3,P1,P4
C
         PARAMETER(P1=32,P4=99)
C
         REAL    ELEN,ALPHA(P1),BETA(P1),FA,F(P4)
C
```

```
      C2 = (1.0/ELEN)**2.0
      C3 = (1.0)/(2.0*ELEN)
      C4 = (1.0)/3.0
      C5 = (1.0)/6.0
C
C     ....initialize the work arrays...............................
      DO 120 IEL =1,NEL
           DO 100 I = 1,6
              DO 90 J= 1,6
                   EKPR(IEL,I,J) = 0.0
                  GAMMA(I,J) = 0.0
                   EKGA(I,J) = 0.0
                 GAEKGA(I,J) = 0.0
                 EK(IEL,I,J) = 0.0
   90         CONTINUE
  100      CONTINUE
C
C     ....calculate the area and inertia terms.....................
       BH = BAVE(IEL)*HAVE(IEL)
      BH3 = BAVE(IEL)*(HAVE(IEL)**3.0)
C
C     ....determine the EKPR matrix...................... .........
      EKPR(IEL,1,1) =  C1*BH
      EKPR(IEL,1,4) = -C1*BH
      EKPR(IEL,2,2) =  C1*C2*BH3
      EKPR(IEL,2,3) =  C1*C3*BH3
      EKPR(IEL,2,5) = -C1*C2*BH3
      EKPR(IEL,2,6) =  C1*C3*BH3
      EKPR(IEL,3,2) =  C1*C3*BH3
      EKPR(IEL,3,3) =  C1*C4*BH3
      EKPR(IEL,3,5) = -C1*C3*BH3
      EKPR(IEL,3,6) =  C1*C5*BH3
      EKPR(IEL,4,1) = -C1*BH
      EKPR(IEL,4,4) =  C1*BH
      EKPR(IEL,5,2) = -C1*C2*BH3
      EKPR(IEL,5,3) = -C1*C3*BH3
      EKPR(IEL,5,5) =  C1*C2*BH3
      EKPR(IEL,5,6) = -C1*C3*BH3
      EKPR(IEL,6,2) =  C1*C3*BH3
      EKPR(IEL,6,3) =  C1*C5*BH3
      EKPR(IEL,6,5) = -C1*C3*BH3
      EKPR(IEL,6,6) =  C1*C4*BH3
C
C     ....determine the GAMMA matrix...............................
         CA = COS(ALPHA(IEL))
         CB = COS(BETA(IEL))
      GAMMA(1,1) =  CA
      GAMMA(1,2) =  CB
      GAMMA(2,1) = -CB
      GAMMA(2,2) =  CA
      GAMMA(3,3) =  1.0
      GAMMA(4,4) =  CA
      GAMMA(4,5) =  CB
      GAMMA(5,4) = -CB
      GAMMA(5,5) =  CA
      GAMMA(6,6) =  1.0

C
C     ....determine the EKGA array.................................
```

```
C       ....form the F-vector.........................................
        F(1) = (ELEN/2.0) * (-COS(BETA(1)))
        F(2) = (ELEN/2.0) * (COS(ALPHA(1)))
        F(3) = 0.0
C
        DO 100 i=2,NEL
           I1 = (i-1)*3 + 1
           I2 = (i-1)*3 + 2
           I3 = (i-1)*3 + 3
C
           F(I1) = (ELEN/2.0)*(-COS(BETA(i)))
     &             +(ELEN/2.0)*(-COS(BETA(i-1)))
           F(I2) = (ELEN/2.0)*(COS(ALPHA(i)))
     &             +(ELEN/2.0)*(COS(ALPHA(i-1)))
           F(I3) = 0.0
  100   CONTINUE
C
        F(NDOF-2) = (ELEN/2.0)*(-COS(BETA(NEL)))
        F(NDOF-1) = (ELEN/2.0)*(COS(ALPHA(NEL)))
        F(NDOF)   = 0.0
C
C       ....scale the F-vector by FA..................................
        DO 200 i=1,NDOF
           F(i) = FA*F(i)
  200   CONTINUE
C
        RETURN
        END
*******************************************************************
*                                                                 *
        SUBROUTINE BNDARY (NDOF,GK,CLAN,FX,FY,FM,F,BX1,BY1,BM1,BX2,
     &                                                      BY2,BM2)
C       ==========================================================
C       This subroutine is used to impose the boundary conditions upon
C       the global stiffness matrix and force vector.
C       ==========================================================
C       ....declare the variables.....................................
        INTEGER NDOF,BX1,BY1,BM1,BX2,BY2,BM2,CLAN,i,N,I1,I2,I3,P4

        PARAMETER(P4=99)

        REAL    GK(P4,P4),FX,FY,FM,F(P4)
C
C       ....invoke the essential boundary conditions..................
        IF (BX1 .EQ. 1) THEN
           CALL IMPOSE_BC (NDOF,GK,1,F)
        ENDIF
C
        IF (BY1 .EQ. 1) THEN
           CALL IMPOSE_BC (NDOF,GK,2,F)
        ENDIF
C
        IF (BM1 .EQ. 1) THEN
           CALL IMPOSE_BC (NDOF,GK,3,F)
        ENDIF
C
        IF (BX2 .EQ. 1) THEN
           N=NDOF-2
           CALL IMPOSE_BC (NDOF,GK,N,F)
        ENDIF
```

```
C
      IF (BY2 .EQ. 1) THEN
         N=NDOF-1
         CALL IMPOSE_BC (NDOF,GK,N,F)
      ENDIF
C
      IF (BM2 .EQ. 1) THEN
         CALL IMPOSE_BC (NDOF,GK,NDOF,F)
      ENDIF
C
C     ....add the concentrated load to the force vector...........
      I1=(CLAN-1)*3+1
      I2=(CLAN-1)*3+2
      I3=(CLAN-1)*3+3
C
      F(I1)=F(I1)+FX
      F(I2)=F(I2)+FY
      F(I3)=F(I3)+FM
C
      RETURN
      END
**********************************************************************
*                                                                    *
      SUBROUTINE IMPOSE_BC (NDOF,GK,N,F)
C     ==============================================================
C     This subroutine is used to do the redundant leg work of impos-
C     ing the boundary conditions.
C     ==============================================================
C     ....declare the variables.....................................
      INTEGER NDOF,N,i,P4

      PARAMETER(P4=99)

      REAL    GK(P4,P4),F(P4)
C
C     ....impose the boundary condition on the GK and F arrays......
      DO 100 i=1,NDOF
         GK(N,i) = 0.0
  100 CONTINUE
         GK(N,N) = 1.0
            F(N) = 0.0
C
      RETURN
      END
**********************************************************************
*                                                                    *
      SUBROUTINE UPSCALE(NDOF,GK,F,BK,BF)
C     ==============================================================
C     This subroutine is used to change the stiffness matrix & force
C     vector from single precision to double precision in order to
C     solve the linear system of equations in double precision.
C     ==============================================================
C     ....declare the variables.....................................
      INTEGER NDOF,i,j,P4

      PARAMETER (P4=99)

      REAL    GK(P4,P4),F(P4)

      REAL*8  BK(P4,P4),BF(P4)
```

```
C
C       ....generate the doubleprecision compliments of GK and F......
        DO 110 i=1,NDOF
           DO 100 j=1,NDOF
              BK(i,j) = GK(i,j)
  100      CONTINUE
              BF(i) = F(i)
  110   CONTINUE
C
        RETURN
        END
**********************************************************************
*                                                                    *
        SUBROUTINE DOWNSCALE(NDOF,BU,U)
C       ==============================================================
C       This subroutine is used to do down scale the double precision
C       solution of the linear system of equations back to single pre-
C       cision. DOT could have problems with double precision numbers!
C       ==============================================================
C       ....declare the variables.....................................
        INTEGER NDOF,i,P4

        PARAMETER (P4=99)

        REAL    U(P4)

        REAL*8  BU(P4)
C
C       ....generate the doubleprecision compliments of GK and F......
        DO 100 i=1,NDOF
           U(i) = BU(i)
  100   CONTINUE
C
        RETURN
        END
**********************************************************************
*                                                                    *
        SUBROUTINE STRESS
C       ==============================================================
C       This subroutine computes the stress at each nodal point.
C       ==============================================================
C       ....declarations..............................................
        INCLUDE 'ARCH_COM.FOR'

        INTEGER     I1,I2,I3,I4,I5,I6

        REAL        CA1,CB1,K1,K2,FPR(P4,6),UPR(6),NORM1,NORM2,
     &              BEND1,BEND2
C
C       ....determine local forces from stiffness and displacement....

        DO 100 i=1, NEL
           I1=(i-1)*3+1
           I2=(i-1)*3+2
           I3=(i-1)*3+3
           I4=(i)*3+1
           I5=(i)*3+2
           I6=(i)*3+3
C
           CB1= COS(BETA(i))
```

```
            CA1= COS(ALPHA(i))
C
                 UPR(1)=  U(I1)*CA1 +  U(I2)*CB1
                 UPR(2)= -U(I1)*CB1 +  U(I2)*CA1
                 UPR(3)=  U(I3)
                 UPR(4)=  U(I4)*CA1 +  U(I5)*CB1
                 UPR(5)= -U(I4)*CB1 +  U(I5)*CA1
                 UPR(6)=  U(I6)
C
              DO 250 L=1,6
                 FPR(i,L)= 0.0
  250         CONTINUE
C
              DO 300 J=1,6
                 DO 350 K=1,6
                    FPR(i,J)= FPR(i,J) + EKPR(i,J,K)*UPR(K)
  350            CONTINUE
  300         CONTINUE
  100    CONTINUE
C        ....determine the bending and normal stresses.................

         SIGMA_N(1) = ABS(FPR(1,1)*(1.0/(BASE(1)*HGT(1))))
         SIGMA_B(1) = ABS(FPR(1,3)*(6.0/(BASE(1)*(HGT(1)**2.0))))
         SIGMA_T(1) = SIGMA_B(1) + SIGMA_N(1)

         DO 400 i=2,NEL

              K1 = 1.0/(BASE(i)*HGT(i))
              K2 = 6.0/(BASE(i)*(HGT(i)**2.0))

            NORM1 = ABS(FPR(i,1)*K1)
            NORM2 = ABS(FPR(i-1,4)*K1)

            BEND1 = ABS(FPR(i,3)*K2)
            BEND2 = ABS(FPR(i-1,6)*K2)

            SIGMA_N(i) = (NORM1+NORM2)/2.0
            SIGMA_B(i) = (BEND1+BEND2)/2.0

            SIGMA_T(i) = SIGMA_B(i) + SIGMA_N(i)

  400    CONTINUE

         SIGMA_N(NSNP) = ABS(FPR(NEL,4)*(1.0/(BASE(NSNP)*HGT(NSNP))))
         SIGMA_B(NSNP) = ABS(FPR(NEL,6)*
     &                   (6.0/(BASE(NSNP)*(HGT(NSNP)**2.0))))
         SIGMA_T(NSNP) = SIGMA_B(NSNP) + SIGMA_N(NSNP)
C
         RETURN
         END
*******************************************************************
*                                                                 *
         SUBROUTINE ARCH_OUTPUT
C        ==========================================================
C        This subroutine formats the final results and output of the
C        optimization problem and stores it in a file named ARCH_OUT.DAT
C        ==========================================================
C        ....declare variables.......................................
         INCLUDE 'ARCH_COM.FOR'
```

```
      REAL    VOL,VOLUME
C
C     ....open output file and write header.........................
      OPEN(9, FILE='ARCH_OUT.DAT', STATUS='UNKNOWN')
C
      WRITE(9,100) LABEL
      WRITE(9,100) ' OPTIMIZATION SOLUTION'
      WRITE(9,105) ' -------------------------------------------
     &----------------------'
  100 FORMAT(/5X,A)
  105 FORMAT(5X,A)
C
C     ....section "A"...............................................
      WRITE(9,100) ' A) Problem Parameters:'
      WRITE(9,110) ' Arch Angle :', ANGLE,  ' Youngs Modulus:',YOUNG
      WRITE(9,110) ' Arch Radius:', RADIUS, ' Yield Strength:',YIELD

      WRITE(9,115) ' No of Design Var:', NDV, ' No of Elements:',NEL
  110 FORMAT(8X,A,F12.3,T38,A,F12.1)
  115 FORMAT(8X,A,I7,T38,A,I10)
C
C     ....section "B"...............................................
      WRITE(9,100) ' B) Derived Constants:'
      WRITE(9,120) ' No of System Nodal Points...',NSNP
      WRITE(9,120) ' No of Degrees of Freedom....',NDOF
      WRITE(9,125) ' Length per Element..........',ELEN
C     WRITE(9,125) ' Phi Angle per Element.......',PHIANG
      WRITE(9,120) ' Number of Iterations........',ITERATE

  120 FORMAT(8X,A,I6)
  125 FORMAT(8X,A,F12.4)
C
C     ....section "C"...............................................
      WRITE(9,100) ' C) Structure Loading:'
      WRITE(9,125) 'FX.........................',FX
      WRITE(9,125) 'FY.........................',FY
      WRITE(9,125) 'FM.........................',FM
      WRITE(9,125) 'FA.........................',FA
C
C     ....section "D"...............................................
      WRITE(9,100) ' D) Elemental Dimensions and Stress Distribution:'
      WRITE(9,210) 'Node','Height','Base','Length','Area'
C
  210 FORMAT(8X,A,T21,A,T36,A,T49,A,T62,A)
  220 FORMAT(8X,I4,T17,F10.5,T32,F10.5,T48,F8.5,T60,F8.5)
      VOLUME = 0.0
C
      DO 300 i=1,NSNP
         AREA = BASE(i)*HGT(i)
         WRITE(9,220) i,HGT(i),BASE(i),ELEN,AREA
  300 CONTINUE
C
C     ....section "E"...............................................
      WRITE(9,100) ' E) Objective Function:'
C
      WRITE(9,310) ' Total structure Volume:',OBJ
  310 FORMAT(/12X,A,F12.6/)
C
      WRITE(9,330) 'Node','Normal Stress','Bending Stress','Total'
      DO 320 i=1,NSNP
```

```
            WRITE(9,340) i,SIGMA_N(i),SIGMA_B(i),SIGMA_T(i)
  320     CONTINUE
  330     FORMAT(8X,A,T18,A,T35,A,T57,A)
  340     FORMAT(8X,I4,T15,F14.1,T32,F14.1,T49,F14.1)
C
C         ....section "F"...........................................
          WRITE(9,100) ' F) Boundary Conditions:'
          WRITE(9,410) 'Node','X-Displ','Y-Displ','Slope'
          WRITE(9,430) 1,BX1,BY1,BM1
          WRITE(9,430) NEL+1,BX2,BY2,BM2
C
C         ....section "G"...........................................
          WRITE(9,100) ' G) Solution Vector:'
          WRITE(9,410) 'Node','X-Displ','Y-Displ','Slope'
          DO 400 i=1,NSNP
                I1=(i-1)*3+1
                I2=(i-1)*3+2
                I3=(i-1)*3+3
            WRITE(9,420) i,U(I1),U(I2),U(I3)
  400     CONTINUE
  410     FORMAT(T9,A,T17,A,T31,A,T46,A)
  420     FORMAT(7X,I5,3E14.6)
  430     FORMAT(7X,I5,T20,I4,T34,I4,T48,I4)
C
          RETURN
          END
```

```
C        ARCH_COMMON
C
C        ....definitions.........................................
C        P1.....The maximum number of elements
C        P2.....The maximum number of global nodal points
C        P3.....The maximum number of design constraints
C        P4.....The maximum number of degrees of freedom
C        p5.....The maximum number of design variables
C
C        ....declare the variables...............................
         INTEGER NEL,NCON,NSNP,NDOF,NDV,METHOD,MINMAX,INFO,IPRINT,
     &           IWK(1000),NRWK,NRIWK,IPRM(20),COUNT,OPTDCS,ITERATE,
     &           PRCSN,CLAN,BX1,BY1,BM1,BX2,BY2,BM2,P1,P2,P3,P4,P5
C
         PARAMETER(P1=32,P2=33,P3=96,P4=99,P5=64)
C
         REAL    ANGLE,RADIUS,ELEN,X(P2),Y(P2),ALPHA(P1),BETA(P1),
     &           YOUNG,YIELD,WK(38800),RPRM(20),OBJ,G(P3),
     &           DV1BG,DV1LO,DV1UP,BASE(P1),BASEL(P1),BASEU(P1),
     &           DV2BG,DV2LO,DV2UP,HGT(P1),HGTL(P1),HGTU(P1),
     &           DESIGN(P5),DESIGNL(P5),DESIGNU(P5),
     &           FA,FX,FY,FM,U(P4),SIGMA_T(P4),SIGMA_N(P4),
     &           SIGMA_B(P4),BAVE(P1),
     &           HAVE(P1),GK(P4,P4),EKPR(P4,6,6)
C
         CHARACTER*30    LABEL
C
C        ....make in common......................................
         COMMON  NEL,NCON,NSNP,NDOF,NDV,METHOD,MINMAX,INFO,IPRINT,IWK,
     &           NRWK,NRIWK,IPRM,COUNT,OPTDCS,ITERATE,PRCSN,CLAN,
     &           BX1,BY1,BM1,BX2,BY2,BM2,
     &           ANGLE,RADIUS,ELEN,X,Y,ALPHA,BETA,YOUNG,YIELD,
     &           WK,RPRM,OBJ,G,DV1BG,DV1LO,DV1UP,BASE,BASEL,BASEU,
     &           DV2BG,DV2LO,DV2UP,HGT,HGTL,HGTU,
     &           DESIGN,DESIGNL,DESIGNU,
     &           FA,FX,FY,FM,U,SIGMA_T,LABEL,SIGMA_N,SIGMA_B,
     &           BAVE,HAVE,GK,EKPR
C
```

APPENDIX D
VALIDATION CASES

Validation #1

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	0.003	Youngs Modulus:	30000000.0
Arch Radius:	1000000.000	Yield Strength:	52000.0
No of Design Var:	10	No of Elements:	4

B) Derived Constants:

No of System Nodal Points...	5
No of Degrees of Freedom....	15
Length per Element..........	11.2500
Number of Iterations........	1

C) Structure Loading:

FX.........................	1000.0000
FY.........................	0.0000
FM.........................	0.0000
FA.........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Node	Height	Base	Length	Area
1	3.00000	1.50000	11.24996	4.50000
2	3.00000	1.50000	11.24996	4.50000
3	3.00000	1.50000	11.24996	4.50000
4	3.00000	1.50000	11.24996	4.50000
5	3.00000	1.50000	11.24996	4.50000

E) Objective Function:

Total structure Volume: 202.499207

Node	Normal Stress	Bending Stress	Total
1	0.0	19999.7	19999.7
2	0.0	14999.7	14999.7
3	0.0	9999.8	9999.8
4	0.0	4999.9	4999.9
5	0.0	0.0	0.0

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	1
5	0	0	0

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.000000E+00
2	0.257807E-01	0.112327E-08	-0.437492E-02
3	0.937478E-01	0.409056E-08	-0.749985E-02
4	0.189839E+00	0.828721E-08	-0.937481E-02
5	0.299993E+00	0.130985E-07	-0.999979E-02

Validation #2

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	0.003	Youngs Modulus:	30000000.0
Arch Radius:	1000000.000	Yield Strength:	52000.0
No of Design Var:	10	No of Elements:	4

B) Derived Constants:

No of System Nodal Points...	5
No of Degrees of Freedom....	15
Length per Element..........	11.2500
Number of Iterations........	1

C) Structure Loading:

FX.........................	0.0000
FY.........................	1000.0000
FM.........................	0.0000
FA.........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Node	Height	Base	Length	Area
1	3.00000	1.50000	11.24996	4.50000
2	3.00000	1.50000	11.24996	4.50000
3	3.00000	1.50000	11.24996	4.50000
4	3.00000	1.50000	11.24996	4.50000
5	3.00000	1.50000	11.24996	4.50000

E) Objective Function:

Total structure Volume: 202.499207

Node	Normal Stress	Bending Stress	Total
1	222.2	0.0	222.2
2	222.2	0.0	222.2
3	222.2	0.0	222.2
4	222.2	0.0	222.2
5	222.2	0.0	222.2

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	1
5	0	0	0

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.000000E+00
2	0.112327E-08	0.833330E-04	-0.191234E-09
3	0.409056E-08	0.166666E-03	-0.327829E-09
4	0.828721E-08	0.249999E-03	-0.409786E-09
5	0.130985E-07	0.333332E-03	-0.437105E-09

Validation #3

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	0.003	Youngs Modulus:	30000000.0
Arch Radius:	1000000.000	Yield Strength:	52000.0
No of Design Var:	10	No of Elements:	4

B) Derived Constants:

No of System Nodal Points...	5
No of Degrees of Freedom....	15
Length per Element..........	11.2500
Number of Iterations........	1

C) Structure Loading:

FX..........................	0.0000
FY..........................	0.0000
FM..........................	10000.0000
FA..........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Node	Height	Base	Length	Area
1	3.00000	1.50000	11.24996	4.50000
2	3.00000	1.50000	11.24996	4.50000
3	3.00000	1.50000	11.24996	4.50000
4	3.00000	1.50000	11.24996	4.50000
5	3.00000	1.50000	11.24996	4.50000

E) Objective Function:

Total structure Volume: 202.499207

Node	Normal Stress	Bending Stress	Total
1	0.0	4444.4	4444.4
2	0.0	4444.4	4444.4
3	0.0	4444.4	4444.4
4	0.0	4444.4	4444.4
5	0.0	4444.4	4444.4

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	1
5	0	0	0

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.000000E+00
2	-0.624985E-02	-0.273190E-09	0.111109E-02
3	-0.249994E-01	-0.109276E-08	0.222218E-02
4	-0.562488E-01	-0.245871E-08	0.333328E-02
5	-0.999979E-01	-0.437105E-08	0.444438E-02

Validation #4

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	90.000	Youngs Modulus:	30000000.0
Arch Radius:	45.000	Yield Strength:	52000.0
No of Design Var:	10	No of Elements:	4

B) Derived Constants:

No of System Nodal Points...	5
No of Degrees of Freedom....	15
Length per Element..........	17.5581
Number of Iterations........	1

C) Structure Loading:

FX.........................	0.0000
FY.........................	1000.0000
FM.........................	0.0000
FA.........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Node	Height	Base	Length	Area
1	3.00000	1.50000	17.55813	4.50000
2	3.00000	1.50000	17.55813	4.50000
3	3.00000	1.50000	17.55813	4.50000
4	3.00000	1.50000	17.55813	4.50000
5	3.00000	1.50000	17.55813	4.50000

E) Objective Function:

Total structure Volume: 316.046356

Node	Normal Stress	Bending Stress	Total
1	217.9	19996.8	20214.8
2	201.4	18475.5	18676.8
3	154.1	14141.1	14295.2
4	83.4	7653.6	7737.0
5	43.3	0.0	43.3

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	1
5	0	0	0

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.000000E+00
2	-0.654528E-01	0.131494E-01	0.750557E-02
3	-0.223473E+00	0.118865E+00	0.138688E-01
4	-0.381495E+00	0.355493E+00	0.181207E-01
5	-0.446951E+00	0.684692E+00	0.196139E-01

Validation #4

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	90.000	Youngs Modulus:	30000000.0
Arch Radius:	45.000	Yield Strength:	52000.0
No of Design Var:	14	No of Elements:	6

B) Derived Constants:

No of System Nodal Points...	7
No of Degrees of Freedom....	21
Length per Element..........	11.7474
Number of Iterations........	1

C) Structure Loading:

FX.........................	0.0000
FY.........................	1000.0000
FM.........................	0.0000
FA.........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Node	Height	Base	Length	Area
1	3.00000	1.50000	11.74736	4.50000
2	3.00000	1.50000	11.74736	4.50000
3	3.00000	1.50000	11.74736	4.50000
4	3.00000	1.50000	11.74736	4.50000
5	3.00000	1.50000	11.74736	4.50000
6	3.00000	1.50000	11.74736	4.50000
7	3.00000	1.50000	11.74736	4.50000

E) Objective Function:

Total structure Volume: 317.178711

Node	Normal Stress	Bending Stress	Total
1	220.3	19988.4	20208.7
2	212.8	19310.1	19522.9
3	190.9	17315.2	17506.0
4	155.9	14139.8	14295.7
5	110.3	9999.2	10109.4
6	57.1	5176.4	5233.4
7	29.0	0.2	29.2

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	1
7	0	0	0

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.000000E+00
2	-0.300317E-01	0.404077E-02	0.512948E-02
3	-0.112085E+00	0.381154E-01	0.991003E-02
4	-0.224178E+00	0.124215E+00	0.140157E-01
5	-0.336278E+00	0.270393E+00	0.171665E-01
6	-0.418343E+00	0.468604E+00	0.191473E-01
7	-0.448382E+00	0.696858E+00	0.198230E-01

Validation #4

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	90.000	Youngs Modulus:	30000000.0
Arch Radius:	45.000	Yield Strength:	52000.0
No of Design Var:	18	No of Elements:	8

B) Derived Constants:

No of System Nodal Points...	9
No of Degrees of Freedom....	27
Length per Element..........	8.8215
Number of Iterations........	1

C) Structure Loading:

FX.........................	0.0000
FY.........................	1000.0000
FM.........................	0.0000
FA.........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Node	Height	Base	Length	Area
1	3.00000	1.50000	8.82154	4.50000
2	3.00000	1.50000	8.82154	4.50000
3	3.00000	1.50000	8.82154	4.50000
4	3.00000	1.50000	8.82154	4.50000
5	3.00000	1.50000	8.82154	4.50000
6	3.00000	1.50000	8.82154	4.50000
7	3.00000	1.50000	8.82154	4.50000
8	3.00000	1.50000	8.82154	4.50000
9	3.00000	1.50000	8.82154	4.50000

E) Objective Function:

Total structure Volume: 317.575592

Node	Normal Stress	Bending Stress	Total
1	221.2	19983.1	20204.3
2	217.0	19602.4	19819.4
3	204.4	18467.7	18672.1
4	184.0	16622.6	16806.6
5	156.6	14138.1	14294.6
6	123.0	11109.3	11232.4
7	84.7	7653.0	7737.7
8	43.2	3901.8	3944.9
9	21.9	0.1	22.0

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	1
9	0	0	0

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.000000E+00
2	-0.170799E-01	0.174758E-02	0.388006E-02
3	-0.657226E-01	0.165686E-01	0.761158E-02
4	-0.138527E+00	0.555485E-01	0.110510E-01
5	-0.224410E+00	0.126097E+00	0.140661E-01
6	-0.310298E+00	0.230817E+00	0.165408E-01
7	-0.383113E+00	0.367110E+00	0.183798E-01
8	-0.431768E+00	0.527570E+00	0.195124E-01
9	-0.448854E+00	0.701109E+00	0.198948E-01

1	0.000000E+00	0.000000E+00	0.000000E+00
2	-0.109773E-01	0.916237E-03	0.311363E-02
3	-0.428386E-01	0.861779E-02	0.615142E-02
4	-0.924699E-01	0.292281E-01	0.903846E-02
5	-0.155017E+00	0.676091E-01	0.117035E-01
6	-0.224357E+00	0.126884E+00	0.140808E-01
7	-0.293705E+00	0.208132E+00	0.161115E-01
8	-0.356268E+00	0.310279E+00	0.177456E-01
9	-0.405923E+00	0.430208E+00	0.189426E-01
10	-0.437804E+00	0.563056E+00	0.196729E-01
11	-0.448790E+00	0.702697E+00	0.199184E-01

Validation #4

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	90.000	Youngs Modulus:	30000000.0
Arch Radius:	45.000	Yield Strength:	52000.0
No of Design Var:	26	No of Elements:	12

B) Derived Constants:

No of System Nodal Points... 13
No of Degrees of Freedom.... 39
Length per Element.......... 5.8863
Number of Iterations........ 1

C) Structure Loading:

FX......................... 0.0000
FY......................... 1000.0000
FM......................... 0.0000
FA......................... 0.0000

D) Elemental Dimensions and Stress Distribution:

Node	Height	Base	Length	Area
1	3.00000	1.50000	5.88628	4.50000
2	3.00000	1.50000	5.88628	4.50000
3	3.00000	1.50000	5.88628	4.50000
4	3.00000	1.50000	5.88628	4.50000
5	3.00000	1.50000	5.88628	4.50000
6	3.00000	1.50000	5.88628	4.50000
7	3.00000	1.50000	5.88628	4.50000
8	3.00000	1.50000	5.88628	4.50000
9	3.00000	1.50000	5.88628	4.50000
10	3.00000	1.50000	5.88628	4.50000
11	3.00000	1.50000	5.88628	4.50000
12	3.00000	1.50000	5.88628	4.50000
13	3.00000	1.50000	5.88628	4.50000

E) Objective Function:

Total structure Volume: 317.859253

Node	Normal Stress	Bending Stress	Total
1	221.7	19973.5	20195.2
2	219.8	19805.9	20025.8
3	214.2	19299.1	19513.3
4	204.9	18461.6	18666.5
5	192.1	17308.0	17500.2
6	176.0	15857.4	16033.4
7	156.9	14135.3	14292.3
8	135.2	12171.0	12306.2
9	111.1	9998.4	10109.5
10	85.0	7653.1	7738.1
11	57.4	5176.2	5233.7
12	29.1	2610.7	2639.8
13	14.7	0.1	14.7

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	1
13	0	0	0

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.000000E+00
2	-0.764866E-02	0.544914E-03	0.260170E-02
3	-0.300750E-01	0.504941E-02	0.515929E-02
4	-0.657529E-01	0.172040E-01	0.762896E-02
5	-0.112252E+00	0.401787E-01	0.996841E-02
6	-0.166406E+00	0.764064E-01	0.121375E-01
7	-0.224523E+00	0.127417E+00	0.140991E-01
8	-0.282643E+00	0.193735E+00	0.158197E-01
9	-0.336806E+00	0.274838E+00	0.172696E-01
10	-0.383318E+00	0.369200E+00	0.184241E-01
11	-0.419010E+00	0.474387E+00	0.192632E-01
12	-0.441447E+00	0.587230E+00	0.197725E-01
13	-0.449100E+00	0.704036E+00	0.199432E-01

Validation #5

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	90.000	Youngs Modulus:	30000000.0
Arch Radius:	32.000	Yield Strength:	52000.0
No of Design Var:	10	No of Elements:	4

B) Derived Constants:

No of System Nodal Points...	5
No of Degrees of Freedom....	15
Length per Element..........	12.4858
Number of Iterations........	1

C) Structure Loading:

FX.........................	0.0000
FY.........................	-5000.0000
FM.........................	0.0000
FA.........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Node	Height	Base	Length	Area
1	3.00000	1.50000	12.48578	4.50000
2	3.00000	1.50000	12.48578	4.50000
3	3.00000	1.50000	12.48578	4.50000
4	3.00000	1.50000	12.48578	4.50000
5	3.00000	1.50000	12.48578	4.50000

E) Objective Function:

Total structure Volume: 224.744095

Node	Normal Stress	Bending Stress	Total
1	1231.2	0.0	1231.2
2	1278.8	12338.0	13516.9
3	1273.2	11971.7	13245.0
4	1073.8	1043.2	2116.9
5	927.6	24725.3	25652.9

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	0
5	1	0	1

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.302544E-02
2	-0.301621E-01	0.547718E-02	0.131377E-02
3	-0.265139E-01	0.237552E-02	-0.205875E-02
4	-0.530910E-02	-0.302736E-01	-0.357488E-02
5	0.000000E+00	-0.589432E-01	0.000000E+00

Validation #5

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	90.000	Youngs Modulus:	30000000.0
Arch Radius:	32.000	Yield Strength:	52000.0
No of Design Var:	14	No of Elements:	6

B) Derived Constants:

No of System Nodal Points...	7
No of Degrees of Freedom....	21
Length per Element..........	8.3537
Number of Iterations........	1

C) Structure Loading:

FX.........................	0.0000
FY.........................	-5000.0000
FM.........................	0.0000
FA.........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Node	Height	Base	Length	Area
1	3.00000	1.50000	8.35368	4.50000
2	3.00000	1.50000	8.35368	4.50000
3	3.00000	1.50000	8.35368	4.50000
4	3.00000	1.50000	8.35368	4.50000
5	3.00000	1.50000	8.35368	4.50000
6	3.00000	1.50000	8.35368	4.50000
7	3.00000	1.50000	8.35368	4.50000

E) Objective Function:

Total structure Volume: 225.549301

Node	Normal Stress	Bending Stress	Total
1	1194.9	0.0	1194.9
2	1247.4	9411.2	10658.6
3	1308.2	13334.9	14643.1
4	1279.8	11503.9	12783.7
5	1164.2	4042.8	5207.1
6	969.3	8539.9	9509.2
7	853.4	25386.8	26240.2

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	0
7	1	0	1

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.316688E-02
2	-0.238606E-01	0.280572E-02	0.229335E-02
3	-0.340200E-01	0.662210E-02	0.182085E-03
4	-0.276224E-01	0.125100E-02	-0.212342E-02
5	-0.128425E-01	-0.185793E-01	-0.356644E-02
6	-0.176538E-02	-0.461115E-01	-0.314903E-02
7	0.000000E+00	-0.613415E-01	0.000000E+00

Validation #5

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	90.000	Youngs Modulus:	30000000.0
Arch Radius:	32.000	Yield Strength:	52000.0
No of Design Var:	18	No of Elements:	8

B) Derived Constants:

No of System Nodal Points...	9
No of Degrees of Freedom....	27
Length per Element..........	6.2731
Number of Iterations........	1

C) Structure Loading:

FX.........................	0.0000
FY.........................	-5000.0000
FM.........................	0.0000
FA.........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Node	Height	Base	Length	Area
1	3.00000	1.50000	6.27310	4.50000
2	3.00000	1.50000	6.27310	4.50000
3	3.00000	1.50000	6.27310	4.50000
4	3.00000	1.50000	6.27310	4.50000
5	3.00000	1.50000	6.27310	4.50000
6	3.00000	1.50000	6.27310	4.50000
7	3.00000	1.50000	6.27310	4.50000
8	3.00000	1.50000	6.27310	4.50000
9	3.00000	1.50000	6.27310	4.50000

E) Objective Function:

Total structure Volume: 225.831528

Node	Normal Stress	Bending Stress	Total
1	1175.4	0.0	1175.4
2	1222.5	7509.3	8731.8
3	1292.3	11997.3	13289.6
4	1312.4	13291.6	14604.0
5	1282.1	11342.3	12624.5
6	1202.5	6224.5	7427.0
7	1076.8	1865.4	2942.1
8	909.6	12616.3	13525.8
9	816.4	25615.1	26431.5

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	0
9	1	0	1

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.321682E-02
2	-0.190172E-01	0.162605E-02	0.269341E-02
3	-0.314948E-01	0.513368E-02	0.133378E-02
4	-0.342107E-01	0.627359E-02	-0.428882E-03
5	-0.280319E-01	0.848441E-03	-0.214589E-02
6	-0.170219E-01	-0.129808E-01	-0.337032E-02
7	-0.654051E-02	-0.331005E-01	-0.367416E-02
8	-0.742209E-03	-0.529373E-01	-0.266477E-02
9	0.000000E+00	-0.622147E-01	0.000000E+00

Validation #5

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	90.000	Youngs Modulus:	30000000.0
Arch Radius:	32.000	Yield Strength:	52000.0
No of Design Var:	22	No of Elements:	10

B) Derived Constants:

No of System Nodal Points... 11
No of Degrees of Freedom.... 33
Length per Element.......... 5.0214
Number of Iterations........ 1

C) Structure Loading:

FX......................... 0.0000
FY......................... -5000.0000
FM......................... 0.0000
FA......................... 0.0000

D) Elemental Dimensions and Stress Distribution:

Node	Height	Base	Length	Area
1	3.00000	1.50000	5.02138	4.50000
2	3.00000	1.50000	5.02138	4.50000
3	3.00000	1.50000	5.02138	4.50000
4	3.00000	1.50000	5.02138	4.50000
5	3.00000	1.50000	5.02138	4.50000
6	3.00000	1.50000	5.02138	4.50000
7	3.00000	1.50000	5.02138	4.50000
8	3.00000	1.50000	5.02138	4.50000
9	3.00000	1.50000	5.02138	4.50000
10	3.00000	1.50000	5.02138	4.50000
11	3.00000	1.50000	5.02138	4.50000

E) Objective Function:

Total structure Volume: 225.962219

Node	Normal Stress	Bending Stress	Total
1	1163.3	0.0	1163.3
2	1204.7	6225.1	7429.8
3	1272.0	10546.0	11817.9
4	1307.9	12856.2	14164.1
5	1311.7	13098.8	14410.6
6	1283.2	11268.0	12551.2
7	1223.1	7408.7	8631.8
8	1132.9	1616.0	2748.9
9	1014.8	5967.4	6982.2
10	871.6	15155.0	16026.7
11	794.2	25720.4	26514.7

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	0
11	1	0	1

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.324004E-02
2	-0.156551E-01	0.103676E-02	0.289272E-02
3	-0.277397E-01	0.372354E-02	0.195701E-02
4	-0.339727E-01	0.607021E-02	0.651332E-03
5	-0.337863E-01	0.569723E-02	-0.796782E-03
6	-0.282253E-01	0.660388E-03	-0.215629E-02
7	-0.195378E-01	-0.983642E-02	-0.319832E-02
8	-0.105138E-01	-0.249420E-01	-0.370184E-02
9	-0.366515E-02	-0.419486E-01	-0.345906E-02
10	-0.355416E-03	-0.564151E-01	-0.228057E-02
11	0.000000E+00	-0.626255E-01	0.000000E+00

Validation #5

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	90.000	Youngs Modulus:	30000000.0
Arch Radius:	32.000	Yield Strength:	52000.0
No of Design Var:	26	No of Elements:	12

B) Derived Constants:

No of System Nodal Points...	13
No of Degrees of Freedom....	39
Length per Element..........	4.1858
Number of Iterations........	1

C) Structure Loading:

FX........................	0.0000
FY........................	-5000.0000
FM........................	0.0000
FA........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Node	Height	Base	Length	Area
1	3.00000	1.50000	4.18580	4.50000
2	3.00000	1.50000	4.18580	4.50000
3	3.00000	1.50000	4.18580	4.50000
4	3.00000	1.50000	4.18580	4.50000
5	3.00000	1.50000	4.18580	4.50000
6	3.00000	1.50000	4.18580	4.50000
7	3.00000	1.50000	4.18580	4.50000
8	3.00000	1.50000	4.18580	4.50000
9	3.00000	1.50000	4.18580	4.50000
10	3.00000	1.50000	4.18580	4.50000
11	3.00000	1.50000	4.18580	4.50000
12	3.00000	1.50000	4.18580	4.50000
13	3.00000	1.50000	4.18580	4.50000

E) Objective Function:

Total structure Volume: 226.033264

Node	Normal Stress	Bending Stress	Total
1	1155.0	0.0	1155.1
2	1191.5	5308.7	6500.2
3	1253.9	9309.8	10563.7
4	1294.8	11935.0	13229.8
5	1313.6	13139.3	14452.9
6	1309.9	12902.2	14212.1
7	1283.8	11227.5	12511.3
8	1235.7	8144.1	9379.9
9	1166.5	3704.7	4871.2
10	1077.3	2015.0	3092.3
11	969.7	8917.1	9886.8
12	845.5	16883.3	17728.8
13	779.5	25777.3	26556.9

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	0
13	1	0	1

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.325259E-02
2	-0.132523E-01	0.707096E-03	0.300569E-02
3	-0.243569E-01	0.274126E-02	0.232580E-02
4	-0.317555E-01	0.506415E-02	0.133772E-02
5	-0.347043E-01	0.631460E-02	0.171546E-03
6	-0.332895E-01	0.514815E-02	-0.103961E-02
7	-0.283308E-01	0.557844E-03	-0.216186E-02
8	-0.211878E-01	-0.785492E-02	-0.306281E-02
9	-0.134844E-01	-0.196869E-01	-0.361389E-02
10	-0.678007E-02	-0.336374E-01	-0.369247E-02
11	-0.221779E-02	-0.475236E-01	-0.318404E-02
12	-0.181929E-03	-0.584105E-01	-0.198409E-02
13	0.000000E+00	-0.628492E-01	0.000000E+00

Validation #5

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	90.000	Youngs Modulus:	30000000.0
Arch Radius:	32.000	Yield Strength:	52000.0
No of Design Var:	30	No of Elements:	14

B) Derived Constants:

No of System Nodal Points...	15
No of Degrees of Freedom....	45
Length per Element..........	3.5885
Number of Iterations........	1

C) Structure Loading:

FX........................	0.0000
FY........................	-5000.0000
FM........................	0.0000
FA........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Node	Height	Base	Length	Area
1	3.00000	1.50000	3.58851	4.50000
2	3.00000	1.50000	3.58851	4.50000
3	3.00000	1.50000	3.58851	4.50000
4	3.00000	1.50000	3.58851	4.50000
5	3.00000	1.50000	3.58851	4.50000
6	3.00000	1.50000	3.58851	4.50000
7	3.00000	1.50000	3.58851	4.50000
8	3.00000	1.50000	3.58851	4.50000
9	3.00000	1.50000	3.58851	4.50000
10	3.00000	1.50000	3.58851	4.50000
11	3.00000	1.50000	3.58851	4.50000
12	3.00000	1.50000	3.58851	4.50000
13	3.00000	1.50000	3.58851	4.50000
14	3.00000	1.50000	3.58851	4.50000
15	3.00000	1.50000	3.58851	4.50000

E) Objective Function:

Total structure Volume: 226.076035

Node	Normal Stress	Bending Stress	Total
1	1149.0	0.0	1149.0
2	1181.5	4624.5	5805.9
3	1238.8	8296.5	9535.3
4	1280.5	10970.1	12250.6
5	1306.1	12611.5	13917.5
6	1315.3	13200.1	14515.4
7	1307.9	12728.6	14036.5
8	1284.1	11202.7	12486.8
9	1244.2	8641.9	9886.1
10	1188.6	5078.2	6266.8
11	1118.0	556.4	1674.5
12	1033.5	4866.7	5900.2
13	935.9	11123.1	12058.9
14	826.5	18133.9	18960.3
15	769.0	25810.9	26579.9

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	0
15	1	0	1

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.326008E-02
2	-0.114679E-01	0.506369E-03	0.307569E-02
3	-0.215484E-01	0.207183E-02	0.256050E-02
4	-0.291563E-01	0.410633E-02	0.179229E-02
5	-0.336353E-01	0.579362E-02	0.852037E-03
6	-0.347835E-01	0.624852E-02	-0.177131E-03
7	-0.328342E-01	0.467242E-02	-0.121096E-02
8	-0.283939E-01	0.495743E-03	-0.216516E-02
9	-0.223430E-01	-0.650287E-02	-0.295641E-02
10	-0.157056E-01	-0.161095E-01	-0.350346E-02
11	-0.949819E-02	-0.276272E-01	-0.372813E-02
12	-0.456627E-02	-0.398712E-01	-0.355627E-02
13	-0.142229E-02	-0.512110E-01	-0.291872E-02
14	-0.946979E-04	-0.596559E-01	-0.175218E-02
15	0.000000E+00	-0.629827E-01	0.000000E+00

Validation #5

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	90.000	Youngs Modulus:	30000000.0
Arch Radius:	32.000	Yield Strength:	52000.0
No of Design Var:	34	No of Elements:	16

B) Derived Constants:

No of System Nodal Points... 17
No of Degrees of Freedom.... 51
Length per Element.......... 3.1403
Number of Iterations........ 1

C) Structure Loading:

FX.......................... 0.0000
FY.......................... -5000.0000
FM.......................... 0.0000
FA.......................... 0.0000

D) Elemental Dimensions and Stress Distribution:

Node	Height	Base	Length	Area
1	3.00000	1.50000	3.14033	4.50000
2	3.00000	1.50000	3.14033	4.50000
3	3.00000	1.50000	3.14033	4.50000
4	3.00000	1.50000	3.14033	4.50000
5	3.00000	1.50000	3.14033	4.50000
6	3.00000	1.50000	3.14033	4.50000
7	3.00000	1.50000	3.14033	4.50000
8	3.00000	1.50000	3.14033	4.50000
9	3.00000	1.50000	3.14033	4.50000
10	3.00000	1.50000	3.14033	4.50000
11	3.00000	1.50000	3.14033	4.50000
12	3.00000	1.50000	3.14033	4.50000
13	3.00000	1.50000	3.14033	4.50000
14	3.00000	1.50000	3.14033	4.50000
15	3.00000	1.50000	3.14033	4.50000
16	3.00000	1.50000	3.14033	4.50000
17	3.00000	1.50000	3.14033	4.50000

E) Objective Function:

Total structure Volume: 226.103821

Node	Normal Stress	Bending Stress	Total
1	1144.4	0.0	1144.4
2	1173.5	4094.6	5268.1
3	1226.1	7465.0	8691.2
4	1266.9	10078.9	11345.9
5	1295.6	11911.1	13206.6
6	1311.7	12943.9	14255.6
7	1315.2	13167.7	14482.9
8	1306.0	12579.8	13885.7
9	1284.2	11186.0	12470.3
10	1250.1	8999.8	10250.0
11	1204.0	6042.2	7246.1
12	1146.2	2341.6	3487.8
13	1077.5	2066.5	3143.9
14	998.3	7139.5	8137.7
15	909.5	12828.8	13738.4
16	812.0	19079.4	19891.4
17	761.1	25831.4	26592.6

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	0
17	1	0	1

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.326459E-02
2	-0.100961E-01	0.376056E-03	0.312172E-02
3	-0.192461E-01	0.160606E-02	0.271838E-02
4	-0.266726E-01	0.333145E-02	0.210622E-02
5	-0.318426E-01	0.503854E-02	0.133893E-02
6	-0.344882E-01	0.613847E-02	0.471678E-03
7	-0.346060E-01	0.604840E-02	-0.439423E-03
8	-0.324379E-01	0.426921E-02	-0.133782E-02
9	-0.284326E-01	0.455661E-03	-0.216707E-02
10	-0.231915E-01	-0.552506E-02	-0.287140E-02
11	-0.174003E-01	-0.135497E-01	-0.339626E-02
12	-0.117521E-01	-0.232130E-01	-0.368879E-02
13	-0.686387E-02	-0.338212E-01	-0.369838E-02
14	-0.319243E-02	-0.444054E-01	-0.337717E-02
15	-0.953191E-03	-0.537569E-01	-0.268042E-02
16	-0.472050E-04	-0.604801E-01	-0.156706E-02
17	0.000000E+00	-0.630648E-01	0.000000E+00

APPENDIX E
CASE STUDIES

OPTIMIZATION #1

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	0.002	Youngs Modulus:	30000000.0
Arch Radius:	1000000.000	Yield Strength:	52000.0
No of Design Var:	18	No of Elements:	8

B) Derived Constants:

No of System Nodal Points...	9
No of Degrees of Freedom....	27
Length per Element..........	4.0000
Number of Iterations........	1

C) Structure Loading:

FX.........................	2000.0000
FY.........................	0.0000
FM.........................	0.0000
FA.........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Node	Height	Base	Length	Area
1	4.19530	0.41953	4.00000	1.76005
2	4.01266	0.40127	4.00000	1.61015
3	3.81169	0.38117	4.00000	1.45290
4	3.58695	0.35869	4.00000	1.28662
5	3.32982	0.33298	4.00000	1.10877
6	3.02540	0.30254	4.00000	0.91530
7	2.64292	0.26429	4.00000	0.69850
8	2.09772	0.20977	4.00000	0.44004
9	0.03041	0.03000	4.00000	0.00091

E) Objective Function:

Total structure Volume: 33.126362

Node	Normal Stress	Bending Stress	Total
1	0.0	52002.4	52002.4
2	0.0	52002.4	52002.4
3	0.0	52002.5	52002.5
4	0.0	52002.4	52002.4
5	0.0	52003.1	52003.1
6	0.0	52000.9	52000.9
7	0.0	52001.6	52001.6
8	0.0	51999.4	51999.4
9	0.1	844.7	844.8

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	1
9	0	0	0

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.000000E+00
2	0.691840E-02	0.295492E-09	-0.338401E-02
3	0.277399E-01	0.119801E-08	-0.693567E-02
4	0.632283E-01	0.274074E-08	-0.106947E-01
5	0.114358E+00	0.496595E-08	-0.147211E-01
6	0.182449E+00	0.793075E-08	-0.191152E-01
7	0.269490E+00	0.117209E-07	-0.240748E-01
8	0.379305E+00	0.165004E-07	-0.301571E-01
9	0.618095E+00	0.268468E-07	-0.744676E-01

OPTIMIZATION #1a

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	0.002	Youngs Modulus:	30000000.0
Arch Radius:	1000000.000	Yield Strength:	52000.0
No of Design Var:	18	No of Elements:	8

B) Derived Constants:

No of System Nodal Points...	9
No of Degrees of Freedom....	27
Length per Element..........	4.0000
Number of Iterations........	1

C) Structure Loading:

FX.........................	2000.0000
FY.........................	0.0000
FM.........................	0.0000
FA.........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Node	Height	Base	Length	Area
1	4.19499	0.41961	4.00000	1.76025
2	4.01244	0.40133	4.00000	1.61030
3	3.81144	0.38124	4.00000	1.45308
4	3.58705	0.35870	4.00000	1.28669
5	3.32942	0.33306	4.00000	1.10891
6	3.02541	0.30254	4.00000	0.91531
7	2.64171	0.26456	4.00000	0.69888
8	2.09811	0.20981	4.00000	0.44021
9	0.10080	0.03000	4.00000	0.00302

E) Objective Function:

Total structure Volume: 33.148262

Node	Normal Stress	Bending Stress	Total
1	0.0	52001.2	52001.2
2	0.0	52001.2	52001.2
3	0.0	52000.0	52000.0
4	0.0	51998.6	51998.6
5	0.0	52003.4	52003.4
6	0.0	52000.0	52000.0
7	0.0	51997.4	51997.4
8	0.0	51970.5	51970.5
9	0.0	0.0	0.0

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	1
9	0	0	0

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.000000E+00
2	0.691870E-02	0.295505E-09	-0.338415E-02
3	0.277409E-01	0.119806E-08	-0.693591E-02
4	0.632300E-01	0.274082E-08	-0.106947E-01
5	0.114360E+00	0.496604E-08	-0.147212E-01
6	0.182452E+00	0.793089E-08	-0.191155E-01
7	0.269496E+00	0.117212E-07	-0.240760E-01
8	0.379315E+00	0.165008E-07	-0.301576E-01
9	0.607042E+00	0.263666E-07	-0.703189E-01

OPTIMIZATION #2

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	90.000	Youngs Modulus:	30000000.0
Arch Radius:	32.000	Yield Strength:	52000.0
No of Design Var:	18	No of Elements:	8

B) Derived Constants:

No of System Nodal Points...	9
No of Degrees of Freedom....	27
Length per Element..........	6.2731
Number of Iterations........	1

C) Structure Loading:

FX.........................	0.0000
FY.........................	-2000.0000
FM.........................	0.0000
FA.........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Node	Height	Base	Length	Area
1	1.95274	1.95611	6.27310	3.81977
2	1.94555	1.93241	6.27310	3.75960
3	1.89934	1.90956	6.27310	3.62691
4	1.92039	1.92025	6.27310	3.68763
5	1.92103	1.91927	6.27310	3.68698
6	1.91825	1.92011	6.27310	3.68326
7	1.91849	1.92476	6.27310	3.69264
8	1.92622	1.91786	6.27310	3.69422
9	1.96640	1.96000	6.27310	3.85413

E) Objective Function:

Total structure Volume: 186.151276

Node	Normal Stress	Bending Stress	Total
1	521.1	51491.0	52012.2
2	519.3	51498.4	52017.7
3	507.0	51507.8	52014.8
4	448.8	45091.9	45540.7
5	381.7	38340.4	38722.1
6	300.0	30197.7	30497.7
7	205.9	20743.2	20949.1
8	104.9	10527.8	10632.7
9	50.9	0.3	51.2

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	1
9	0	0	0

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.000000E+00
2	0.345895E-01	-0.351714E-02	-0.110489E-01
3	0.134858E+00	-0.340464E-01	-0.222560E-01
4	0.287683E+00	-0.115848E+00	-0.328233E-01
5	0.469409E+00	-0.265100E+00	-0.419063E-01
6	0.651576E+00	-0.487184E+00	-0.493731E-01
7	0.806243E+00	-0.776659E+00	-0.549243E-01
8	0.909657E+00	-0.111768E+01	-0.583234E-01
9	0.945957E+00	-0.148635E+01	-0.594191E-01

OPTIMIZATION #2a

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	90.000	Youngs Modulus:	30000000.0
Arch Radius:	32.000	Yield Strength:	52000.0
No of Design Var:	18	No of Elements:	8

B) Derived Constants:

No of System Nodal Points...	9
No of Degrees of Freedom....	27
Length per Element..........	6.2731
Number of Iterations........	1

C) Structure Loading:

FX.........................	0.0000
FY.........................	-2000.0000
FM.........................	0.0000
FA.........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Node	Height	Base	Length	Area
1	3.91339	0.49197	6.27310	1.92527
2	4.10780	0.43837	6.27310	1.80073
3	4.11506	0.41151	6.27310	1.69337
4	3.97207	0.39721	6.27310	1.57773
5	3.69253	0.39033	6.27310	1.44130
6	2.42308	0.70760	6.27310	1.71458
7	2.31581	0.53330	6.27310	1.23503
8	1.30739	0.84851	6.27310	1.10933
9	0.80670	1.49956	6.27310	1.20969

E) Objective Function:

Total structure Volume: 77.775108

Node	Normal Stress	Bending Stress	Total
1	1033.8	50968.7	52002.6
2	1084.1	50916.9	52000.9
3	1085.9	50913.5	51999.4
4	1048.9	50949.6	51998.5
5	976.5	51020.7	51997.2
6	645.0	51350.6	51995.6
7	616.7	51379.1	51995.9
8	350.0	51653.2	52003.2
9	162.0	0.2	162.2

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	1
9	0	0	0

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.000000E+00
2	0.165765E-01	-0.185682E-02	-0.530039E-02
3	0.640268E-01	-0.164901E-01	-0.104791E-01
4	0.136735E+00	-0.556094E-01	-0.157515E-01
5	0.226861E+00	-0.129851E+00	-0.213260E-01
6	0.324976E+00	-0.249654E+00	-0.277857E-01
7	0.421348E+00	-0.430238E+00	-0.369135E-01
8	0.499870E+00	-0.689424E+00	-0.482073E-01
9	0.534109E+00	-0.103740E+01	-0.595043E-01

OPTIMIZATION #3

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	90.000	Youngs Modulus:	30000000.0
Arch Radius:	32.000	Yield Strength:	52000.0
No of Design Var:	18	No of Elements:	8

B) Derived Constants:

No of System Nodal Points...	9
No of Degrees of Freedom....	27
Length per Element..........	6.2731
Number of Iterations........	1

C) Structure Loading:

FX.........................	2000.0000
FY.........................	0.0000
FM.........................	0.0000
FA.........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Node	Height	Base	Length	Area
1	3.66465	0.55100	6.27310	2.01923
2	3.83344	0.40649	6.27310	1.55825
3	3.34200	0.41258	6.27310	1.37883
4	2.46546	0.54860	6.27310	1.35256
5	1.99742	0.55572	6.27310	1.11001
6	1.48940	0.58247	6.27310	0.86752
7	1.21374	0.66160	6.27310	0.80301
8	0.58386	0.48072	6.27310	0.28068
9	0.77051	1.86386	6.27310	1.43612

E) Objective Function:

Total structure Volume: 56.657707

Node	Normal Stress	Bending Stress	Total
1	97.1	51895.5	51992.6
2	249.2	51745.2	51994.4
3	552.4	51445.1	51997.4
4	817.5	51180.4	51997.9
5	1267.9	50730.6	51998.5
6	1907.7	50087.6	51995.3
7	2290.0	29991.8	32281.8
8	6955.2	45025.1	51980.3
9	1386.0	0.0	1386.0

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	1
9	0	0	0

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.000000E+00
2	0.185814E-01	-0.180717E-02	-0.574511E-02
3	0.720148E-01	-0.179297E-01	-0.117837E-01
4	0.158405E+00	-0.639459E-01	-0.190296E-01
5	0.275821E+00	-0.160028E+00	-0.286801E-01
6	0.416853E+00	-0.331363E+00	-0.409667E-01
7	0.559677E+00	-0.597638E+00	-0.537583E-01
8	0.678467E+00	-0.986551E+00	-0.722173E-01
9	0.725130E+00	-0.145498E+01	-0.764554E-01

OPTIMIZATION #3a

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	90.000	Youngs Modulus:	30000000.0
Arch Radius:	32.000	Yield Strength:	52000.0
No of Design Var:	18	No of Elements:	8

B) Derived Constants:

No of System Nodal Points...	9
No of Degrees of Freedom....	27
Length per Element..........	6.2731
Number of Iterations........	2

C) Structure Loading:

FX..........................	2000.0000
FY..........................	0.0000
FM..........................	0.0000
FA..........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Node	Height	Base	Length	Area
1	4.19827	0.41983	6.27310	1.76255
2	3.90887	0.39089	6.27310	1.52793
3	3.58581	0.35858	6.27310	1.28581
4	3.22371	0.32237	6.27310	1.03923
5	2.81849	0.28185	6.27310	0.79439
6	2.36310	0.23631	6.27310	0.55843
7	1.84434	0.18443	6.27310	0.34016
8	0.99509	0.18101	6.27310	0.18012
9	0.60305	0.06348	6.27310	0.03828

E) Objective Function:

Total structure Volume: 41.341122

Node	Normal Stress	Bending Stress	Total
1	111.2	51891.6	52002.9
2	254.1	51749.1	52003.2
3	592.4	51411.0	52003.3
4	1064.0	50938.9	52002.9
5	1771.6	50231.5	52003.1
6	2963.6	49040.2	52003.8
7	5406.0	46591.3	51997.2
8	10838.0	41165.8	52003.8
9	51995.7	0.0	51995.7

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	1
9	0	0	0

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.000000E+00
2	0.173605E-01	-0.168480E-02	-0.536752E-02
3	0.677523E-01	-0.168806E-01	-0.111584E-01
4	0.148066E+00	-0.596161E-01	-0.175022E-01
5	0.251532E+00	-0.144153E+00	-0.246090E-01
6	0.367672E+00	-0.284910E+00	-0.328323E-01
7	0.482192E+00	-0.497395E+00	-0.428618E-01
8	0.577701E+00	-0.806931E+00	-0.575020E-01
9	0.627437E+00	-0.126845E+01	-0.822389E-01

OPTIMIZATION #4

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	90.000	Youngs Modulus:	30000000.0
Arch Radius:	32.000	Yield Strength:	52000.0
No of Design Var:	18	No of Elements:	8

B) Derived Constants:

No of System Nodal Points...	9
No of Degrees of Freedom....	27
Length per Element..........	6.2731
Number of Iterations........	2

C) Structure Loading:

FX.........................	0.0000
FY.........................	-2000.0000
FM.........................	1000.0000
FA.........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Node	Height	Base	Length	Area
1	4.20406	0.42041	6.27310	1.76741
2	4.17639	0.41764	6.27310	1.74422
3	4.09209	0.40921	6.27310	1.67452
4	3.94749	0.39475	6.27310	1.55827
5	3.73436	0.37344	6.27310	1.39455
6	3.43725	0.34373	6.27310	1.18147
7	3.02033	0.30203	6.27310	0.91224
8	1.95373	0.35108	6.27310	0.68592
9	0.79249	0.21196	6.27310	0.16798

E) Objective Function:

Total structure Volume: 63.252686

Node	Normal Stress	Bending Stress	Total
1	1126.2	50868.9	51995.1
2	1119.2	50874.8	51994.1
3	1098.2	50896.4	51994.6
4	1062.0	50929.1	51991.1
5	1009.3	50986.3	51995.6
6	936.0	51055.6	51991.6
7	835.0	51156.8	51991.8
8	566.1	51424.8	51990.9
9	1166.5	45071.7	46238.2

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	1
9	0	0	0

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.000000E+00
2	0.158777E-01	-0.180201E-02	-0.507744E-02
3	0.619007E-01	-0.160076E-01	-0.102264E-01
4	0.133298E+00	-0.544288E-01	-0.155283E-01
5	0.222312E+00	-0.127764E+00	-0.210892E-01
6	0.318467E+00	-0.245256E+00	-0.270698E-01
7	0.408886E+00	-0.414819E+00	-0.337708E-01
8	0.479103E+00	-0.646809E+00	-0.425065E-01
9	0.512886E+00	-0.990889E+00	-0.605568E-01

OPTIMIZATION #4a

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	90.000	Youngs Modulus:	30000000.0
Arch Radius:	32.000	Yield Strength:	52000.0
No of Design Var:	18	No of Elements:	8

B) Derived Constants:

No of System Nodal Points...	9
No of Degrees of Freedom....	27
Length per Element..........	6.2731
Number of Iterations........	2

C) Structure Loading:

FX.........................	0.0000
FY.........................	-2000.0000
FM.........................	10000.0000
FA.........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Node	Height	Base	Length	Area
1	3.99646	0.39965	6.27310	1.59717
2	3.96579	0.39658	6.27310	1.57275
3	3.87202	0.38720	6.27310	1.49925
4	3.70940	0.37094	6.27310	1.37596
5	3.46539	0.34654	6.27310	1.20089
6	3.11238	0.31124	6.27310	0.96869
7	2.57625	0.25763	6.27310	0.66371
8	1.43829	0.14383	6.27310	0.20687
9	2.25936	0.22769	6.27310	0.51444

E) Objective Function:

Total structure Volume: 53.206200

Node	Normal Stress	Bending Stress	Total
1	1246.2	50757.2	52003.4
2	1241.2	50761.6	52002.8
3	1226.5	50776.2	52002.8
4	1202.8	50799.4	52002.1
5	1172.0	50828.7	52000.7
6	1141.6	50859.8	52001.4
7	1147.6	50851.9	51999.5
8	1877.1	50127.0	52004.0
9	381.0	51622.1	52003.1

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	1
9	0	0	0

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.000000E+00
2	0.166829E-01	-0.190698E-02	-0.533238E-02
3	0.650796E-01	-0.168603E-01	-0.107525E-01
4	0.140297E+00	-0.573560E-01	-0.163633E-01
5	0.234385E+00	-0.134897E+00	-0.223074E-01
6	0.336635E+00	-0.259876E+00	-0.288281E-01
7	0.434064E+00	-0.442670E+00	-0.365050E-01
8	0.515013E+00	-0.710561E+00	-0.496258E-01
9	0.544307E+00	-0.100920E+01	-0.415951E-01

OPTIMIZATION #5

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	90.000	Youngs Modulus:	30000000.0
Arch Radius:	32.000	Yield Strength:	52000.0
No of Design Var:	18	No of Elements:	8

B) Derived Constants:

No of System Nodal Points...	9
No of Degrees of Freedom....	27
Length per Element..........	6.2731
Number of Iterations........	2

C) Structure Loading:

FX.........................	0.0000
FY.........................	0.0000
FM.........................	0.0000
FA.........................	-100.0000

D) Elemental Dimensions and Stress Distribution:

Node	Height	Base	Length	Area
1	4.94233	0.49484	6.27310	2.44566
2	4.59883	0.46036	6.27310	2.11711
3	4.20711	0.42104	6.27310	1.77135
4	3.76835	0.37698	6.27310	1.42060
5	3.27617	0.32787	6.27310	1.07416
6	2.72309	0.27231	6.27310	0.74152
7	2.08293	0.20953	6.27310	0.43644
8	1.31418	0.13235	6.27310	0.17394
9	0.14293	0.03000	6.27310	0.00429

E) Objective Function:

Total Structure Volume: 55.704273

Node	Normal Stress	Bending Stress	Total
1	1173.9	50828.3	52002.2
2	1210.8	50791.8	52002.6
3	1109.9	50893.1	52002.9
4	996.4	51006.4	52002.7
5	868.5	51135.7	52004.2
6	724.0	51279.7	52003.7
7	555.5	51446.8	52002.3
8	351.8	51646.5	51998.3
9	1.8	4.8	6.6

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	1
9	0	0	0

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.000000E+00
2	0.144353E-01	-0.168651E-02	-0.447172E-02
3	0.564283E-01	-0.146790E-01	-0.932896E-02
4	0.123657E+00	-0.508635E-01	-0.147199E-01
5	0.210848E+00	-0.122671E+00	-0.208707E-01
6	0.309650E+00	-0.243321E+00	-0.281898E-01
7	0.408546E+00	-0.428622E+00	-0.375918E-01
8	0.492877E+00	-0.706927E+00	-0.522128E-01
9	0.557215E+00	-0.136017E+0[illegible]	-0.130850E+00

OPTIMIZATION #6

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	90.000	Youngs Modulus:	30000000.0
Arch Radius:	32.000	Yield Strength:	52000.0
No of Design Var:	18	No of Elements:	8

B) Derived Constants:

No of System Nodal Points...	9
No of Degrees of Freedom....	27
Length per Element..........	6.2731
Number of Iterations........	1

C) Structure Loading:

FX.........................	0.0000
FY.........................	-2000.0000
FM.........................	0.0000
FA.........................	-100.0000

D) Elemental Dimensions and Stress Distribution:

Node	Height	Base	Length	Area
1	5.82316	0.58232	6.27310	3.39091
2	5.56534	0.55653	6.27310	3.09731
3	5.25381	0.52538	6.27310	2.76025
4	4.88828	0.48883	6.27310	2.38953
5	4.46201	0.44620	6.27310	1.99096
6	3.96199	0.39620	6.27310	1.56974
7	3.35911	0.33591	6.27310	1.12836
8	2.27093	0.32719	6.27310	0.74303
9	0.40176	0.51850	6.27310	0.20831

E) Objective Function:

Total structure Volume: 97.474487

Node	Normal Stress	Bending Stress	Total
1	1433.6	50561.4	51995.1
2	1457.9	50537.3	51995.2
3	1378.4	50617.0	51995.5
4	1284.9	50710.9	51995.8
5	1175.4	50821.4	51996.9
6	1046.4	50951.4	51997.8
7	889.9	51109.2	51999.1
8	605.0	51393.6	51998.6
9	940.6	0.6	941.2

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	1
9	0	0	0

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.000000E+00
2	0.118390E-01	-0.148107E-02	-0.371830E-02
3	0.461727E-01	-0.122074E-01	-0.763781E-02
4	0.100311E+00	-0.414621E-01	-0.118319E-01
5	0.169098E+00	-0.982482E-01	-0.164007E-01
6	0.244936E+00	-0.191026E+00	-0.215055E-01
7	0.317850E+00	-0.327870E+00	-0.274519E-01
8	0.375826E+00	-0.519533E+00	-0.353806E-01
9	0.404874E+00	-0.815207E+00	-0.533505E-01

OPTIMIZATION #6a

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	90.000	Youngs Modulus:	30000000.0
Arch Radius:	32.000	Yield Strength:	52000.0
No of Design Var:	18	No of Elements:	8

B) Derived Constants:

No of System Nodal Points...	9
No of Degrees of Freedom....	27
Length per Element..........	6.2731
Number of Iterations........	1

C) Structure Loading:

FX.........................	0.0000
FY.........................	-2000.0000
FM.........................	0.0000
FA.........................	-100.0000

D) Elemental Dimensions and Stress Distribution:

Node	Height	Base	Length	Area
1	5.82323	0.58232	6.27310	3.39101
2	5.56545	0.55654	6.27310	3.09742
3	5.25394	0.52539	6.27310	2.76039
4	4.88843	0.48884	6.27310	2.38968
5	4.46221	0.44622	6.27310	1.99114
6	3.96197	0.39623	6.27310	1.56986
7	2.96078	0.43150	6.27310	1.27759
8	2.01471	0.41514	6.27310	0.83639
9	0.53831	1.20792	6.27310	0.65023

E) Objective Function:

Total structure Volume: 101.764938

Node	Normal Stress	Bending Stress	Total
1	1433.6	50556.2	51989.7
2	1457.8	50531.4	51989.2
3	1378.3	50609.9	51988.2
4	1284.9	50703.3	51988.1
5	1175.3	50812.4	51987.8
6	1046.3	50946.8	51993.1
7	786.1	51211.2	51997.3
8	537.6	51463.1	52000.6
9	301.4	0.5	302.0

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	1
9	0	0	0

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.000000E+00
2	0.118375E-01	-0.148090E-02	-0.371783E-02
3	0.461666E-01	-0.122058E-01	-0.763674E-02
4	0.100297E+00	-0.414562E-01	-0.118301E-01
5	0.169072E+00	-0.982332E-01	-0.163980E-01
6	0.244899E+00	-0.190995E+00	-0.215019E-01
7	0.318251E+00	-0.328633E+00	-0.277224E-01
8	0.377833E+00	-0.525529E+00	-0.367195E-01
9	0.404775E+00	-0.799478E+00	-0.474619E-01

OPTIMIZATION #7

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	90.000	Youngs Modulus:	30000000.0
Arch Radius:	32.000	Yield Strength:	52000.0
No of Design Var:	18	No of Elements:	8

B) Derived Constants:

No of System Nodal Points...	9
No of Degrees of Freedom....	27
Length per Element..........	6.2731
Number of Iterations........	1

C) Structure Loading:

FX..........................	0.0000
FY..........................	-8000.0000
FM..........................	0.0000
FA..........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Node	Height	Base	Length	Area
1	0.49766	0.32385	6.27310	0.16117
2	2.49460	0.47039	6.27310	1.17344
3	3.05960	0.46175	6.27310	1.41278
4	2.96505	0.48983	6.27310	1.45238
5	2.41709	0.49892	6.27310	1.20593
6	0.96211	0.15996	6.27310	0.15390
7	3.08578	0.43109	6.27310	1.33023
8	4.00244	0.54499	6.27310	2.18129
9	4.21890	0.79543	6.27310	3.35583

E) Objective Function:

Total structure Volume: 64.558678

Node	Normal Stress	Bending Stress	Total
1	51998.6	0.2	51998.8
2	7361.8	44637.4	51999.2
3	6359.0	45640.2	51999.3
4	6185.7	45813.3	51999.0
5	7163.7	44836.1	51999.7
6	51734.2	267.2	52001.4
7	5246.1	46199.3	51445.4
8	2625.6	49374.5	52000.1
9	1501.8	50497.5	51999.4

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	0
9	1	0	1

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.307764E-01
2	-0.149671E+00	0.117776E-01	0.102322E-01
3	-0.192288E+00	0.232035E-01	0.336248E-02
4	-0.193966E+00	0.225997E-01	-0.298150E-02
5	-0.161968E+00	-0.546921E-02	-0.100998E-01
6	-0.785744E-01	-0.112048E+00	-0.273030E-01
7	-0.161085E-01	-0.234507E+00	-0.111216E-01
8	-0.133252E-02	-0.285889E+00	-0.514754E-02
9	0.000000E+00	-0.303321E+00	0.000000E+00

OPTIMIZATION #7a

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	90.000	Youngs Modulus:	30000000.0
Arch Radius:	32.000	Yield Strength:	52000.0
No of Design Var:	18	No of Elements:	8

B) Derived Constants:

No of System Nodal Points...	9
No of Degrees of Freedom....	27
Length per Element..........	6.2731
Number of Iterations........	2

C) Structure Loading:

FX.........................	0.0000
FY.........................	-8000.0000
FM.........................	0.0000
FA.........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Node	Height	Base	Length	Area
1	0.67370	0.23920	6.27310	0.16115
2	2.61798	0.42995	6.27310	1.12561
3	3.27094	0.40726	6.27310	1.33212
4	3.15268	0.43628	6.27310	1.37546
5	2.37931	0.51326	6.27310	1.22121
6	0.77194	0.19791	6.27310	0.15277
7	3.03132	0.44320	6.27310	1.34348
8	4.32486	0.46885	6.27310	2.02772
9	4.62704	0.66330	6.27310	3.06910

E) Objective Function:

Total structure Volume: 61.674786

Node	Normal Stress	Bending Stress	Total
1	52005.8	0.1	52005.8
2	7674.6	44327.7	52002.3
3	6743.9	45258.6	52002.5
4	6531.3	45470.1	52001.4
5	7073.5	44927.7	52001.1
6	52110.1	1128.4	53238.5
7	5193.7	46613.6	51807.3
8	2824.0	49177.3	52001.2
9	1641.8	50360.2	52002.1

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	0
9	1	0	1

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.285876E-01
2	-0.140675E+00	0.106575E-01	0.102757E-01
3	-0.184847E+00	0.224797E-01	0.386082E-02
4	-0.190542E+00	0.239361E-01	-0.204050E-02
5	-0.163888E+00	0.229189E-03	-0.886111E-02
6	-0.790007E-01	-0.108136E+00	-0.284664E-01
7	-0.148789E-01	-0.233588E+00	-0.104461E-01
8	-0.115786E-02	-0.281574E+00	-0.472253E-02
9	0.000000E+00	-0.297573E+00	0.000000E+00

OPTIMIZATION #8

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	90.000	Youngs Modulus:	30000000.0
Arch Radius:	32.000	Yield Strength:	52000.0
No of Design Var:	18	No of Elements:	8

B) Derived Constants:

No of System Nodal Points...	9
No of Degrees of Freedom....	27
Length per Element..........	6.2731
Number of Iterations........	2

C) Structure Loading:

FX..........................	0.0000
FY..........................	-8000.0000
FM..........................	0.0000
FA..........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Node	Height	Base	Length	Area
1	3.96647	0.39665	6.27310	1.57329
2	2.66054	0.26605	6.27310	0.70784
3	2.69799	0.26980	6.27310	0.72791
4	3.29169	0.32917	6.27310	1.08352
5	3.29281	0.32928	6.27310	1.08426
6	2.70298	0.27030	6.27310	0.73061
7	2.65240	0.26524	6.27310	0.70352
8	3.95112	0.39511	6.27310	1.56114
9	4.87425	0.48694	6.27310	2.37347

E) Objective Function:

Total structure Volume: 52.992058

Node	Normal Stress	Bending Stress	Total
1	5469.7	46526.9	51996.6
2	12833.6	39153.1	51986.7
3	13542.6	38456.9	51999.5
4	9462.4	42532.7	51995.1
5	9456.6	42533.2	51989.9
6	13496.0	38480.4	51976.4
7	12918.1	39108.3	52026.4
8	5103.2	46906.0	52009.2
9	3085.5	48816.0	51901.5

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	1
9	1	0	1

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.000000E+00
2	-0.237838E-01	0.695637E-03	0.631554E-02
3	-0.683462E-01	0.113025E-01	0.624288E-02
4	-0.894876E-01	0.199189E-01	0.336519E-03
5	-0.792890E-01	0.896548E-02	-0.506634E-02
6	-0.479138E-01	-0.329894E-01	-0.109648E-01
7	-0.147824E-01	-0.100892E+00	-0.110874E-01
8	-0.821552E-03	-0.152604E+00	-0.471284E-02
9	0.000000E+00	-0.168974E+00	0.000000E+00

OPTIMIZATION #8a

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	90.000	Youngs Modulus:	30000000.0
Arch Radius:	32.000	Yield Strength:	52000.0
Arch Height:	2.000	No of Elements:	12

B) Derived Constants:

No of System Nodal Points...	13
No of Degrees of Freedom....	39
Length per Element..........	4.1858
Number of Iterations........	1

C) Structure Loading:

FX..........................	0.0000
FY..........................	-8000.0000
FM..........................	0.0000
FA..........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Element	Height	Base	Length	Volume
1	2.00000	1.72707	4.18580	14.45838
2	2.00000	0.94577	4.18580	7.91762
3	2.00000	0.27194	4.18580	2.27661
4	2.00000	0.65040	4.18580	5.44488
5	2.00000	1.09075	4.18580	9.13130
6	2.00000	0.78986	4.18580	6.61236
7	2.00000	1.06223	4.18580	8.89255
8	2.00000	0.75388	4.18580	6.31116
9	2.00000	0.25849	4.18580	2.16400
10	2.00000	0.93275	4.18580	7.80861
11	2.00000	1.75263	4.18580	14.67236
12	2.00000	2.65143	4.18580	22.19675

E) Objective Function:

Total structure Volume: 107.886574

Node	Stress
1	51991.36
2	49986.21
3	46231.32
4	51991.33
5	49404.85
6	51991.04
7	51841.79
8	42080.21
9	51997.34
10	51279.07
11	52434.15
12	51908.94
13	51935.26

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	1
13	1	0	1

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.000000E+00
2	-0.121854E-01	0.457452E-03	0.527930E-02
3	-0.442765E-01	0.615066E-02	0.947556E-02
4	-0.861533E-01	0.177060E-01	0.907372E-02
5	-0.112986E+00	0.297132E-01	0.435047E-02
6	-0.121058E+00	0.342979E-01	-0.109393E-03
7	-0.110454E+00	0.237526E-01	-0.701848E-02
8	-0.850803E-01	-0.622361E-02	-0.115516E-01
9	-0.537900E-01	-0.547404E-01	-0.155000E-01
10	-0.252856E-01	-0.118485E+00	-0.155501E-01
11	-0.744947E-02	-0.173190E+00	-0.110976E-01
12	-0.654979E-03	-0.209063E+00	-0.580765E-02
13	0.000000E+00	-0.222069E+00	0.000000E+00

OPTIMIZATION #9

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	180.000	Youngs Modulus:	30000000.0
Arch Radius:	32.000	Yield Strength:	52000.0
No of Design Var:	18	No of Elements:	8

B) Derived Constants:

No of System Nodal Points...	9
No of Degrees of Freedom....	27
Length per Element..........	12.4858
Number of Iterations........	2

C) Structure Loading:

FX..........................	16000.0000
FY..........................	0.0000
FM..........................	0.0000
FA..........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Node	Height	Base	Length	Area
1	3.87125	2.61830	12.48578	10.13610
2	2.99788	1.94284	12.48578	5.82439
3	0.78829	0.32181	12.48578	0.25368
4	2.77540	1.34286	12.48578	3.72695
5	2.82891	1.32326	12.48578	3.74337
6	2.47737	1.02727	12.48578	2.54492
7	1.60172	1.17975	12.48578	1.88963
8	0.98586	0.84212	12.48578	0.83021
9	1.27732	1.14827	12.48578	1.46670

E) Objective Function:

Total structure Volume: 287.147583

Node	Normal Stress	Bending Stress	Total
1	573.5	51426.4	51999.9
2	1458.0	50542.0	52000.0
3	51241.7	770.7	52012.4
4	4166.3	47833.6	52000.0
5	2239.2	49761.4	52000.5
6	404.8	51595.6	52000.5
7	1007.1	50991.3	51998.4
8	2995.1	49002.5	51997.6
9	1835.2	0.1	1835.2

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	1
9	0	1	0

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.000000E+00
2	0.904933E-01	-0.176853E-01	-0.130647E-01
3	0.558262E+00	-0.327630E+00	-0.608843E-01
4	0.883004E+00	-0.806151E+00	-0.171659E-01
5	0.909118E+00	-0.928170E+00	-0.266783E-02
6	0.923784E+00	-0.854074E+00	0.134924E-01
7	0.109988E+01	-0.590018E+00	0.348143E-01
8	0.169034E+01	-0.194612E+00	0.717773E-01
9	0.266363E+01	0.000000E+00	0.833529E-01

OPTIMIZATION #9a

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	180.000	Youngs Modulus:	30000000.0
Arch Radius:	32.000	Yield Strength:	52000.0
No of Design Var:	18	No of Elements:	8

B) Derived Constants:

No of System Nodal Points...	9
No of Degrees of Freedom....	27
Length per Element...........	12.4858
Number of Iterations........	2

C) Structure Loading:

FX.........................	16000.0000
FY.........................	0.0000
FM.........................	0.0000
FA.........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Node	Height	Base	Length	Area
1	1.07555	0.19099	12.48578	0.20542
2	5.49600	0.69864	12.48578	3.83971
3	5.93999	0.96376	12.48578	5.72472
4	5.98177	1.04035	12.48578	6.22316
5	6.00000	0.54928	12.48578	3.29571
6	5.57273	0.58244	12.48578	3.24580
7	4.24805	0.49093	12.48578	2.08549
8	2.17557	0.52420	12.48578	1.14044
9	0.24362	0.66203	12.48578	0.16128

E) Objective Function:

Total structure Volume: 344.338989

Node	Normal Stress	Bending Stress	Total
1	53383.0	1.3	53384.3
2	3451.4	50161.1	53612.5
3	2907.2	50644.1	53551.3
4	2812.1	50764.5	53576.5
5	2854.3	77667.4	80521.7
6	925.1	52414.6	53339.7
7	2660.2	50774.2	53434.4
8	6355.6	47117.6	53473.2
9	48642.3	0.0	48642.3

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	0
9	0	1	0

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	-0.532524E-01
2	0.538764E+00	-0.103983E+00	-0.253326E-01
3	0.766928E+00	-0.254801E+00	-0.178851E-01
4	0.867834E+00	-0.403592E+00	-0.108023E-01
5	0.885116E+00	-0.482743E+00	-0.246003E-02
6	0.891319E+00	-0.450538E+00	0.696965E-02
7	0.974611E+00	-0.324620E+00	0.161255E-01
8	0.122814E+01	-0.153178E+00	0.301513E-01
9	0.197499E+01	0.000000E+00	0.765027E-01

OPTIMIZATION #10

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	180.000	Youngs Modulus:	30000000.0
Arch Radius:	32.000	Yield Strength:	52000.0
No of Design Var:	18	No of Elements:	8

B) Derived Constants:

No of System Nodal Points...	9
No of Degrees of Freedom....	27
Length per Element..........	12.4858
Number of Iterations........	2

C) Structure Loading:

FX.........................	16000.0000
FY.........................	0.0000
FM.........................	0.0000
FA.........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Node	Height	Base	Length	Area
1	3.87807	2.57526	12.48578	9.98702
2	3.14748	1.72693	12.48578	5.43547
3	0.88930	0.29131	12.48578	0.25906
4	2.92467	1.14515	12.48578	3.34919
5	2.96833	1.05559	12.48578	3.13335
6	2.03171	1.03518	12.48578	2.10318
7	0.77820	0.60572	12.48578	0.47137
8	1.64419	0.87326	12.48578	1.43580
9	1.68701	1.15400	12.48578	1.94680

E) Objective Function:

Total structure Volume: 256.608276

Node	Normal Stress	Bending Stress	Total
1	631.7	51368.8	52000.5
2	1646.5	50354.2	52000.7
3	51530.6	479.7	52010.3
4	4692.9	47307.6	52000.5
5	2706.4	49294.1	52000.5
6	579.8	51421.7	52001.5
7	4780.6	47222.8	52003.4
8	2050.8	49951.2	52002.0
9	1637.1	50364.7	52001.8

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	1
9	0	1	1

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.000000E+00
2	0.882631E-01	-0.172024E-01	-0.127257E-01
3	0.520760E+00	-0.303339E+00	-0.559400E-01
4	0.823131E+00	-0.747606E+00	-0.172292E-01
5	0.850524E+00	-0.874578E+00	-0.359896E-02
6	0.865174E+00	-0.800410E+00	0.136811E-01
7	0.115273E+01	-0.368882E+00	0.570417E-01
8	0.165293E+01	-0.331472E-01	0.251769E-01
9	0.181555E+01	0.000000E+00	0.000000E+00

OPTIMIZATION #11

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	180.000	Youngs Modulus:	30000000.0
Arch Radius:	32.000	Yield Strength:	52000.0
No of Design Var:	18	No of Elements:	8

B) Derived Constants:

No of System Nodal Points...	9
No of Degrees of Freedom....	27
Length per Element..........	12.4858
Number of Iterations........	2

C) Structure Loading:

FX.........................	0.0000
FY.........................	-12000.0000
FM.........................	1000.0000
FA.........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Node	Height	Base	Length	Area
1	1.15824	1.02865	12.48578	1.19142
2	2.00490	1.12219	12.48578	2.24987
3	2.04000	1.08818	12.48578	2.21989
4	0.57320	0.21096	12.48578	0.12092
5	2.42160	1.36445	12.48578	3.30415
6	1.23847	0.70722	12.48578	0.87587
7	1.54335	1.08778	12.48578	1.67882
8	1.43388	0.95977	12.48578	1.37619
9	1.49848	1.26079	12.48578	1.88926

E) Objective Function:

Total structure Volume: 153.076752

Node	Normal Stress	Bending Stress	Total
1	5944.7	0.0	5944.7
2	3277.7	48720.7	51998.4
3	3321.9	48527.9	51849.8
4	51700.0	253.5	51953.4
5	1618.5	50381.3	51999.8
6	6817.0	45180.4	51997.4
7	4083.4	47917.5	52000.9
8	4865.8	47133.8	51999.6
9	3360.4	48642.0	52002.3

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	0
9	1	1	1

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.456664E-01
2	-0.471806E+00	0.920810E-01	0.241675E-01
3	-0.619634E+00	0.189140E+00	0.415893E-02
4	-0.359468E+00	-0.206478E+00	-0.589599E-01
5	-0.269622E+00	-0.667966E+00	0.495288E-02
6	-0.213488E+00	-0.379826E+00	0.343967E-01
7	0.128494E-01	-0.370866E-01	0.214467E-01
8	0.888700E-01	0.160224E-01	-0.531602E-02
9	0.000000E+00	0.000000E+00	0.000000E+00

OPTIMIZATION #11a

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	180.000	Youngs Modulus:	30000000.0
Arch Radius:	32.000	Yield Strength:	52000.0
No of Design Var:	18	No of Elements:	8

B) Derived Constants:

No of System Nodal Points... 9
No of Degrees of Freedom.... 27
Length per Element.......... 12.4858
Number of Iterations........ 2

C) Structure Loading:

FX......................... 0.0000
FY......................... -24000.0000
FM......................... 1000.0000
FA......................... 0.0000

D) Elemental Dimensions and Stress Distribution:

Node	Height	Base	Length	Area
1	1.25459	1.11170	12.48578	1.39472
2	2.56424	1.39293	12.48578	3.57180
3	2.61780	1.33869	12.48578	3.50443
4	0.76364	0.31775	12.48578	0.24265
5	3.03972	1.74821	12.48578	5.31405
6	1.59975	0.89444	12.48578	1.43088
7	1.98431	1.34883	12.48578	2.67651
8	1.85250	1.19860	12.48578	2.22040
9	1.96692	1.46917	12.48578	2.88973

E) Objective Function:

Total structure Volume: 241.778809

Node	Normal Stress	Bending Stress	Total
1	10133.6	0.0	10133.7
2	4120.0	47880.2	52000.2
3	4199.2	47797.9	51997.1
4	51413.5	611.5	52025.0
5	2009.1	49990.9	52000.0
6	8340.7	43657.1	51997.9
7	5124.9	46875.8	52000.8
8	6039.8	45960.0	51999.8
9	4402.2	47599.3	52001.5

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	0
9	1	1	1

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.377959E-01
2	-0.377866E+00	0.726540E-01	0.168604E-01
3	-0.474151E+00	0.134826E+00	0.149644E-02
4	-0.277207E+00	-0.167482E+00	-0.440847E-01
5	-0.210637E+00	-0.513900E+00	0.340668E-02
6	-0.168729E+00	-0.295879E+00	0.262287E-01
7	0.359865E-02	-0.330045E-01	0.166200E-01
8	0.647848E-01	0.107673E-01	-0.365121E-02
9	0.000000E+00	0.000000E+00	0.000000E+00

OPTIMIZATION #12

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	180.000	Youngs Modulus:	30000000.0
Arch Radius:	32.000	Yield Strength:	52000.0
No of Design Var:	18	No of Elements:	8

B) Derived Constants:

No of System Nodal Points...	9
No of Degrees of Freedom....	27
Length per Element..........	12.4858
Number of Iterations........	2

C) Structure Loading:

FX.........................	-9000.0000
FY.........................	-17000.0000
FM.........................	1000.0000
FA.........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Node	Height	Base	Length	Area
1	2.07490	1.64551	12.48578	3.41427
2	0.81408	0.49069	12.48578	0.39946
3	2.06842	0.85360	12.48578	1.76560
4	2.08744	0.85702	12.48578	1.78898
5	1.11662	0.11166	12.48578	0.12468
6	2.82593	0.84646	12.48578	2.39204
7	3.62188	1.18265	12.48578	4.28341
8	0.81741	0.25531	12.48578	0.20869
9	2.18518	1.96045	12.48578	4.28395

E) Objective Function:

Total structure Volume: 156.554611

Node	Normal Stress	Bending Stress	Total
1	1331.2	50668.2	51999.4
2	13159.1	38839.6	51998.8
3	3525.9	48473.0	51998.9
4	3491.6	48507.9	51999.4
5	42640.2	6458.5	49098.6
6	1495.5	50503.6	51999.1
7	1368.2	50631.9	52000.2
8	51935.8	45.6	51981.4
9	2914.9	49085.0	52000.0

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	1
9	1	1	1

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.000000E+00
2	-0.385756E+00	0.754815E-01	0.480384E-01
3	-0.777271E+00	0.334000E+00	0.140370E-01
4	-0.808788E+00	0.378437E+00	-0.538832E-02
5	-0.737499E+00	0.351551E-02	-0.430591E-01
6	-0.812780E+00	-0.364503E+00	-0.423870E-02
7	-0.800801E+00	-0.345997E+00	0.959411E-02
8	-0.415861E+00	-0.859055E-01	0.510140E-01
9	0.000000E+00	0.000000E+00	0.000000E+00

OPTIMIZATION #13

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	180.000	Youngs Modulus:	30000000.0
Arch Radius:	32.000	Yield Strength:	52000.0
No of Design Var:	18	No of Elements:	8

B) Derived Constants:

No of System Nodal Points...	9
No of Degrees of Freedom....	27
Length per Element..........	12.4858
Number of Iterations........	2

C) Structure Loading:

FX..........................	16000.0000
FY..........................	0.0000
FM..........................	1000.0000
FA..........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Node	Height	Base	Length	Area
1	3.88833	2.61929	12.48578	10.18466
2	3.06710	1.88559	12.48578	5.78328
3	0.92232	0.26590	12.48578	0.24524
4	2.98689	1.21410	12.48578	3.62638
5	3.00467	1.32694	12.48578	3.98700
6	1.88382	0.97124	12.48578	1.82964
7	0.67362	0.55665	12.48578	0.37497
8	1.49367	0.78025	12.48578	1.16544
9	1.67967	1.18504	12.48578	1.99048

E) Objective Function:

Total structure Volume: 265.960205

Node	Normal Stress	Bending Stress	Total
1	536.3	51465.6	52001.9
2	1412.3	50589.8	52002.0
3	51994.3	96.5	52090.8
4	4244.8	47757.2	52002.1
5	3935.9	48065.6	52001.5
6	4523.6	47478.1	52001.8
7	4413.4	47572.7	51986.1
8	1855.2	50144.8	52000.0
9	1175.7	36538.5	37714.2

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	1
9	0	1	1

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.000000E+00
2	0.892425E-01	-0.174556E-01	-0.128954E-01
3	0.527041E+00	-0.307447E+00	-0.566398E-01
4	0.833359E+00	-0.758307E+00	-0.176816E-01
5	0.862231E+00	-0.894399E+00	-0.436349E-02
6	0.880363E+00	-0.814806E+00	0.140516E-01
7	0.117817E+01	-0.368085E+00	0.590398E-01
8	0.168336E+01	-0.291622E-01	0.222087E-01
9	0.182676E+01	0.000000E+00	0.000000E+00

OPTIMIZATION #13a

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	180.000	Youngs Modulus:	30000000.0
Arch Radius:	32.000	Yield Strength:	52000.0
No of Design Var:	18	No of Elements:	8

B) Derived Constants:

No of System Nodal Points... 9
No of Degrees of Freedom.... 27
Length per Element.......... 12.4858
Number of Iterations........ 2

C) Structure Loading:

FX.......................... 16000.0000
FY.......................... 0.0000
FM.......................... 1000.0000
FA.......................... 0.0000

D) Elemental Dimensions and Stress Distribution:

Node	Height	Base	Length	Area
1	5.58951	1.27357	12.48578	7.11864
2	5.10116	0.69418	12.48578	3.54110
3	1.56517	0.15652	12.48578	0.24497
4	4.38397	0.58589	12.48578	2.56854
5	4.63376	0.58228	12.48578	2.69813
6	3.27838	0.34568	12.48578	1.13328
7	1.61894	0.16189	12.48578	0.26210
8	2.57960	0.25796	12.48578	0.66544
9	2.91537	0.29154	12.48578	0.84994

E) Objective Function:

Total structure Volume: 175.647415

Node	Normal Stress	Bending Stress	Total
1	765.7	51234.1	51999.8
2	2303.6	49696.1	51999.8
3	52019.3	211.4	52230.6
4	5991.5	46008.2	51999.7
5	5816.0	46183.7	51999.7
6	7301.1	44698.3	51999.4
7	6284.3	35672.9	41957.2
8	3234.0	48765.9	51999.9
9	2740.5	47790.4	50530.9

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	1
9	0	1	1

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.000000E+00
2	0.563288E-01	-0.107647E-01	-0.813259E-02
3	0.306688E+00	-0.174214E+00	-0.318503E-01
4	0.481161E+00	-0.425410E+00	-0.977707E-02
5	0.497296E+00	-0.493441E+00	-0.126135E-02
6	0.513184E+00	-0.431268E+00	0.955349E-02
7	0.666860E+00	-0.199686E+00	0.294584E-01
8	0.932019E+00	-0.202692E-01	0.147632E-01
9	0.102734E+01	0.000000E+00	0.000000E+00

OPTIMIZATION #14

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	180.000	Youngs Modulus:	30000000.0
Arch Radius:	32.000	Yield Strength:	52000.0
No of Design Var:	18	No of Elements:	8

B) Derived Constants:

No of System Nodal Points...	9
No of Degrees of Freedom....	27
Length per Element..........	12.4858
Number of Iterations........	2

C) Structure Loading:

FX.........................	9000.0000
FY.........................	-5000.0000
FM.........................	1000.0000
FA.........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Node	Height	Base	Length	Area
1	2.36812	1.89649	12.48578	4.49111
2	0.84905	0.08490	12.48578	0.07209
3	3.71044	0.71004	12.48578	2.63458
4	3.05720	0.50168	12.48578	1.53373
5	1.56758	0.15986	12.48578	0.25059
6	2.11368	0.60867	12.48578	1.28654
7	2.28870	0.73028	12.48578	1.67139
8	2.09919	0.65663	12.48578	1.37838
9	0.85662	0.28601	12.48578	0.24500

E) Objective Function:

Total structure Volume: 121.283012

Node	Normal Stress	Bending Stress	Total
1	251.3	48722.1	48973.4
2	18883.4	33152.8	52036.2
3	422.4	51577.1	51999.5
4	831.1	51168.1	51999.1
5	9637.3	42361.6	51998.9
6	2478.0	49521.0	51998.9
7	2079.4	49920.1	51999.5
8	2346.2	49653.4	51999.6
9	12165.1	0.2	12165.3

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	1
9	1	1	0

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.000000E+00
2	0.427757E+00	-0.853866E-01	-0.525055E-01
3	0.819383E+00	-0.346183E+00	-0.786212E-02
4	0.822885E+00	-0.351655E+00	0.527751E-02
5	0.767984E+00	-0.809962E-01	0.313859E-01
6	0.828458E+00	0.231809E+00	0.108871E-01
7	0.841128E+00	0.252532E+00	-0.809329E-02
8	0.653208E+00	0.128115E+00	-0.271114E-01
9	0.000000E+00	0.000000E+00	-0.664133E-01

OPTIMIZATION #15a

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	180.000	Youngs Modulus:	30000000.0
Arch Radius:	32.000	Yield Strength:	52000.0
No of Design Var:	18	No of Elements:	8

B) Derived Constants:

No of System Nodal Points...	9
No of Degrees of Freedom....	27
Length per Element..........	12.4858
Number of Iterations........	2

C) Structure Loading:

FX.........................	0.0000
FY.........................	32000.0000
FM.........................	1000.0000
FA.........................	100.0000

D) Elemental Dimensions and Stress Distribution:

Node	Height	Base	Length	Area
1	3.25280	2.04372	12.48578	6.64780
2	3.20982	1.57865	12.48578	5.06717
3	1.05585	0.53193	12.48578	0.56164
4	3.45172	1.93749	12.48578	6.68769
5	4.53938	2.40127	12.48578	10.90026
6	3.87991	2.04928	12.48578	7.95104
7	2.87337	1.82061	12.48578	5.23130
8	1.78269	1.33703	12.48578	2.38351
9	1.43268	1.44887	12.48578	2.07577

E) Objective Function:

Total structure Volume: 516.579224

Node	Normal Stress	Bending Stress	Total
1	3235.7	48783.9	52019.6
2	3969.0	48027.9	51996.9
3	28718.2	2630.5	31348.8
4	1520.8	50476.2	51997.0
5	574.6	51424.1	51998.7
6	1021.3	50976.6	51997.8
7	2359.0	49638.0	51997.0
8	6361.0	45636.9	51998.0
9	7781.0	0.1	7781.1

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	1
9	0	1	0

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.000000E+00
2	-0.800741E-01	0.174874E-01	0.125050E-01
3	-0.427959E+00	0.254098E+00	0.443218E-01
4	-0.654608E+00	0.596950E+00	0.100603E-01
5	-0.666896E+00	0.660397E+00	-0.108817E-02
6	-0.682870E+00	0.578787E+00	-0.114177E-01
7	-0.811784E+00	0.384649E+00	-0.243721E-01
8	-0.118582E+01	0.132794E+00	-0.440123E-01
9	-0.183804E+01	0.000000E+00	-0.579555E-01

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